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The Law of Economic Interdependencies

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(work in progress)

The building blocks

Axiomatic restrictions of any trade network

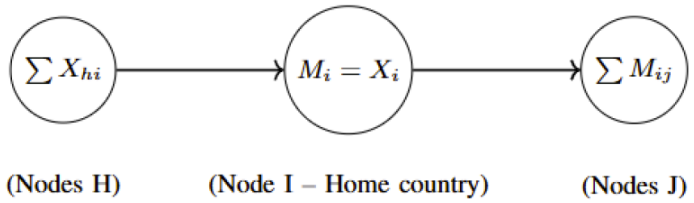
$$(1)\quad \sum_{i=1}^{n} X_i = \sum_{i=1}^{n} M_i$$

Global trade balance

$$(2)\quad \sum_{j=1}^{n} X_{ij} = X_i$$

$$(3)\quad \sum_{j=1}^{n} M_{ij} = M_i$$

Flow consistency



Inflow-outflow condition

$$\frac{\Delta M_t}{M_t} = \frac{\Delta Y_t}{Y_t} \cdot \frac{\Delta M_t / M_t}{\Delta Y_t / Y_t}$$

International Engel's law identity

$$\theta_i = \sum_{j=1}^{n} w_{ij} \cdot \varepsilon_j$$

Mirrored income-elasticity transformation

$$\boxed{\forall i, j \in V\colon \exists\ directed\ path\ i \rightsquigarrow j}$$

Strongly connected network
(ensures Perron-Frobenius theorem)

Antonio Serra: "*A Short Treatise on the Causes that Can Make Kingdoms Abound in Gold and Silver even in the Absence of Mines*" (1613)

BREVE TRATTATO
DELLE CAVSE,
CHE POSSONO FAR ABBONDARE
Li Regni d'oro, & argento.
DOVE NON SONO MINIERE
Con applicatione al Regno di Napoli.
DEL DOTTOR ANTONIO SERRA,
della Città di Cosenza.
DIVISO IN TRE PARTI.

IN NAPOLI,
Appresso Lazzaro Scorriggio. M.DC.XIII.
CON LICENZA DE' SVPERIORI.

Antonio Serra: "*A Short Treatise on the Causes that Can Make Kingdoms Abound in Gold and Silver even in the Absence of Mines*" (1613)

''[…] trade in commodities that are **imported** to meet the country's needs will produce a **shortage of money**, not an abundance.''

"[…] **manufactured goods** generally yield much higher earnings than agricultural produce."

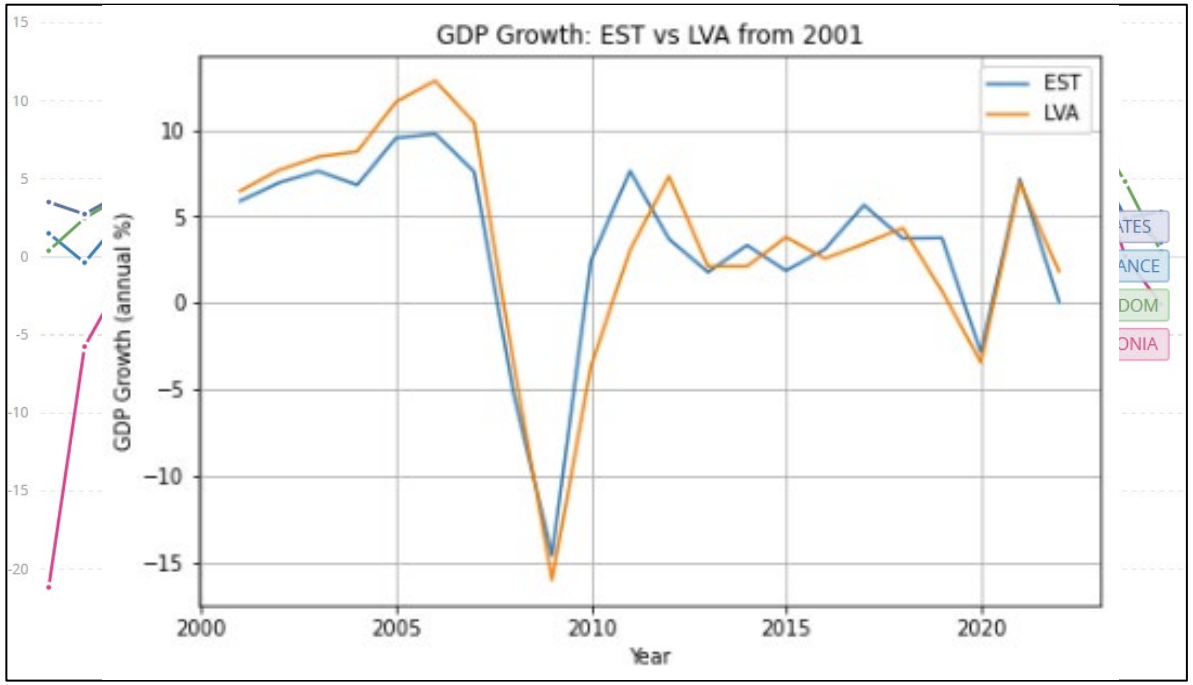
"In Italy the first place with respect to this accident [multiple manufactures] is held by the **city of Venice**, […] so that a large quantity of goods are produced by the manufacturers and consequently **large amounts of money** […] **enter the city in exchange for them**. In the city of Naples, on the other hand, this accident is scarcely present at all, for most of the manufacturing activities are lacking […]

In the first place, **there is no woollen artisanate in the Kingdom [of Naples]** making fine fabrics, and as a consequence clothes are imported. The Kingdom has to import all medicines […], **almost all of them from Venice**. […] All chemical substances, both artificial and natural, have to be imported – chiefly from Venice.

"To begin with Venice, since an outflow of money necessarily presupposes an inflow (for without it, an outflow would be impossible), **the difficulty consists in finding an inflow which is not only sufficient to compensate for the outflow** but exceeds it to such an extent that it produces the abundance of money which in fact exists."

Motivation: output synchronization

This paper argues the reason why we observe output synchronization (GDP co-movements) in the international trade network, is a mechanical outcome of extending **Thirlwall's law (1979)** as a system of simultaneous equations or **network eigenvector problem**. The eigenvector could be understood as the **global foreign-trade multiplier**.



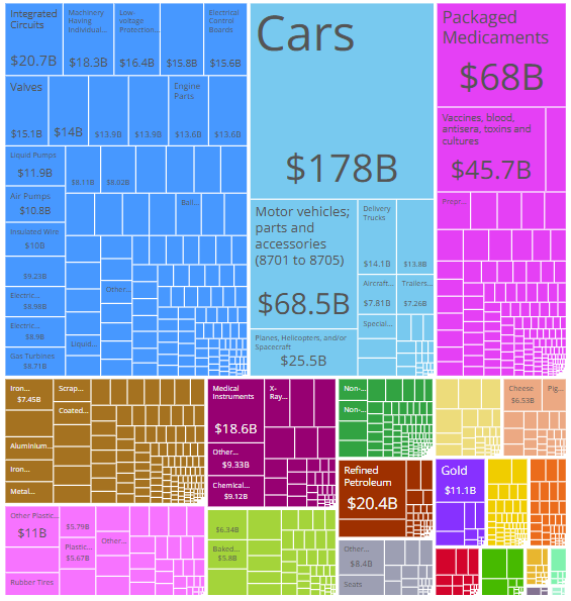
GDP growth (US, France, UK, Estonia)
Source: World Bank

GDP growth (Zambia, Chile, Peru)
Source: World Bank

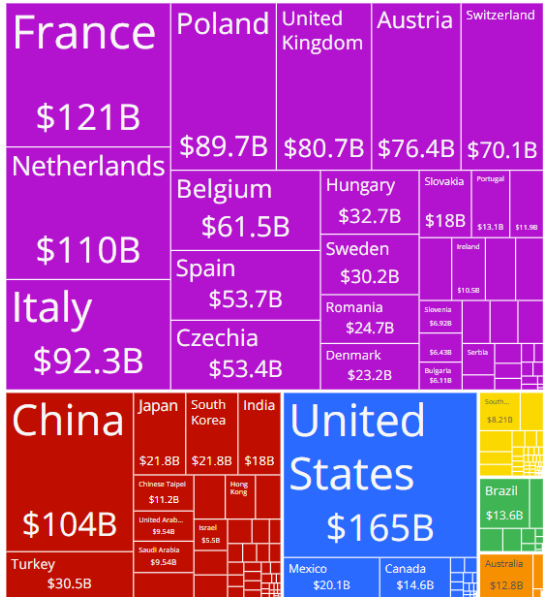
Output synchronization: empirical evidence

Output synchronization remains one of the **main challenges in macroeconomics**, but standard (IRBC) models fall short in replicating its empirical outcomes (Kollmann, 2017).
Despite theoretical shortcomings, **empirical evidence shows strong predictors** of cross-country output co-movement:

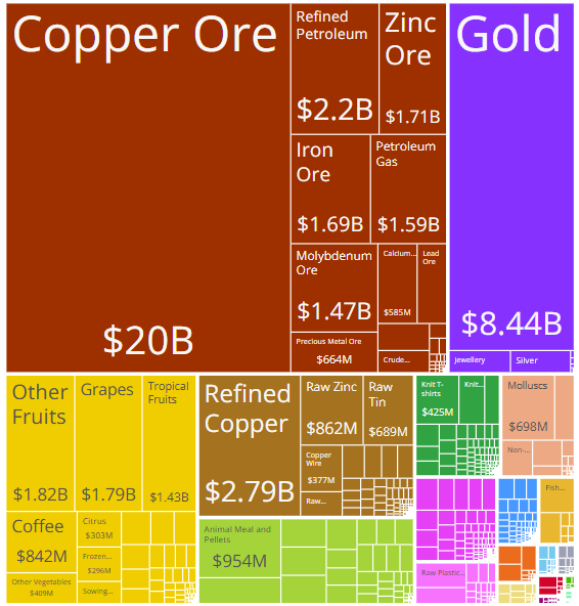
- Depth and type of trade integration (intra-industry trade)
- Similarity in production structures



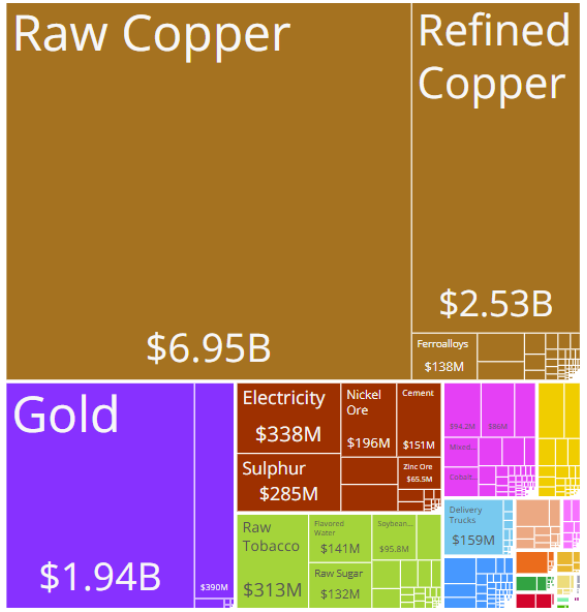
Germany – Exports (2023)
Source: oec.world



Germany – Export destinations (2023)
Source: oec.world



Peru – Exports (2023)
Source: oec.world



Zambia – Exports (2023)
Source: oec.world

Income elasticities of demand in structuralism

The Engel's law in international trade

Higher quality products have **higher income elasticities**.
(*Bils and Klenow, 2001; Chen and Juvenal, 2018*)

"With income elasticity differences across sectors, the **expenditure shares** are more skewed towards **higher income elastic sectors** in richer countries [. . .]"
(*Matsuyama, 2019*)

"[…] **relative productivity changes** across sectors respond endogenously to **changes in the relative market sizes** caused by economic growth due to Engel's Law"
(Matsuyama, 2019)

High-tech production (high-income elastic) is usually more effective at meeting higher and increasing foreign demand

Thirlwall's Law (1979) and Krugman's 45-degree rule (1988)

Serra (1613) "proto foreign-trade multiplier"

Harrod (1933) static trade-multiplier

Prebisch (1959) income elasticity of import demand – center-periphery dynamic

Houthakker and Magee (1969) empirical income elasticities-growth correlation

"ECLAC school" Latin-American structuralists

Thirlwall (1979) balance of payment's constrained growth

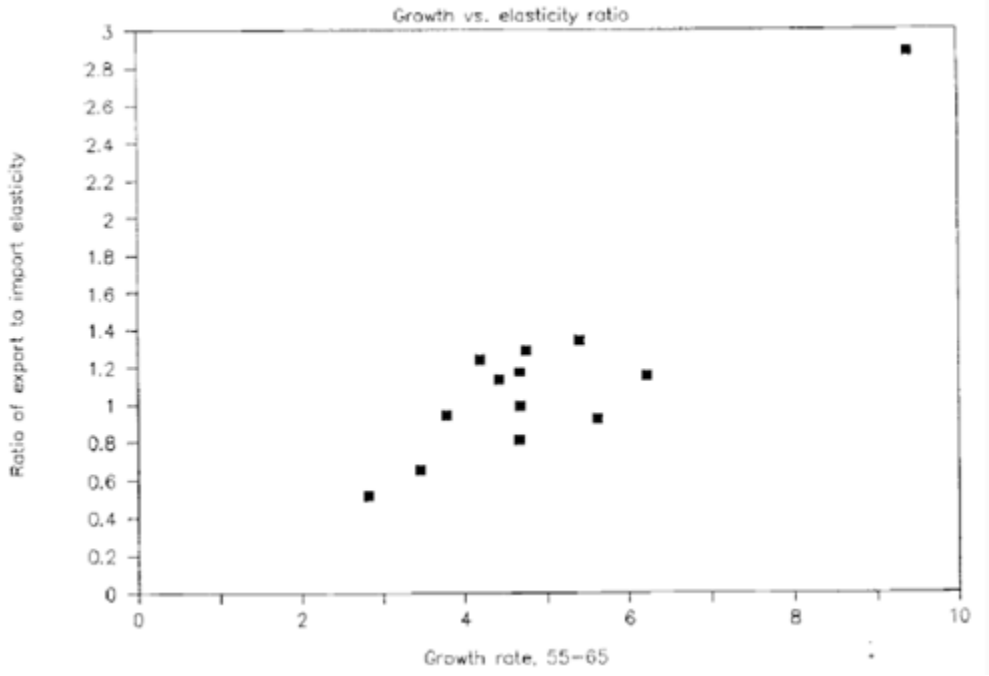
Krugman (1988) 45-degree rule: income-elasticities are endogenous to growth, they just ''follow'' growth rates (spurious correlation).

Wu (2005) – Recognizes Houthakker and Magee's elasticities, but aligns with Krugman

$$\widehat{Y}_i = \widehat{Y}_j \cdot \frac{\varepsilon_i}{\theta_i}$$

Krugman's 45-degree rule (1988)

[...] the model predicts a **spurious relationship** between relative income elasticities and relative growth rates. This is because **fast-growing countries are able to occupy a larger share of the world market** by expanding the range of goods they produce at a faster rate then the rest of the world. **As a result, they appear to face** high income elasticities of demand for their exports, while having low income elasticities of import demand.



Response: Thirlwall's law is not a growth-accounting equation, it just describes relative growth rates.

Krugman's a-priori assumption (1988)

"**I am simply going to dismiss a priori** the argument that income elasticities determine economic growth, rather than the other way around. [...] we all know that differences in growth rates among countries are primarily determined by differences in the rate of growth of **total factor productivity** [...] it is hard to see what channel links balance of payments due to unfavorable income elasticities to total factor productivity growth."

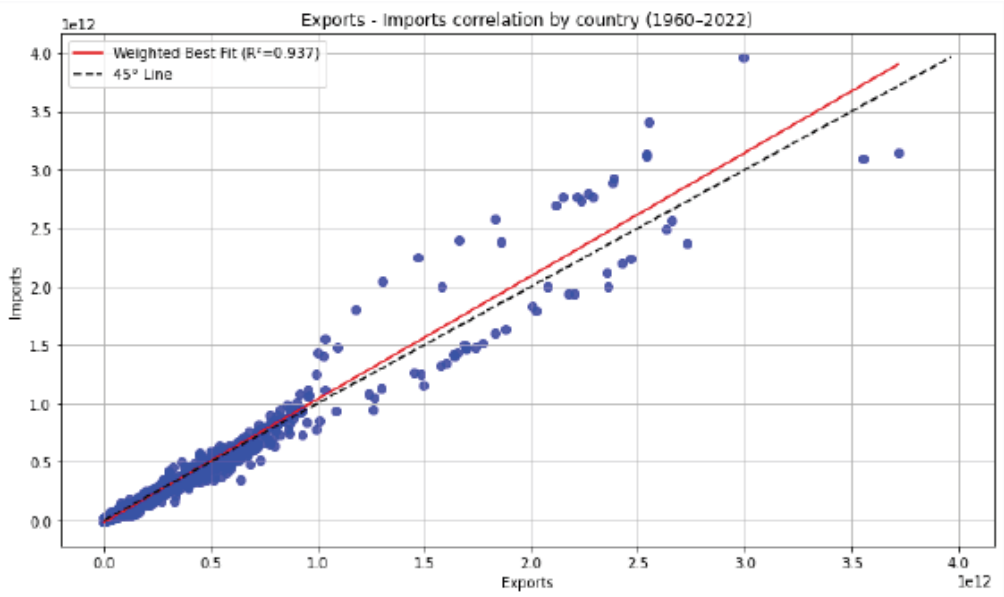
Shaikh (1974) and Felipe and McCombie's (2001, 2005, 2010) demonstration

$$Y = wL + rK \quad \equiv \quad Y = AK^{\alpha}L^{\beta}$$

Here, A arises solely as the integration constant; **it has no independent technological interpretation** beyond ensuring the level of Y matches the accounting identity in the base period. The demonstration emphasizes that A does not represent an independent "technology index" or "efficiency residual" in the sense used by Solow. Instead, **it reflects whatever proportion of output growth cannot be explained by input growth under fixed shares**, which is itself driven by accounting conventions.

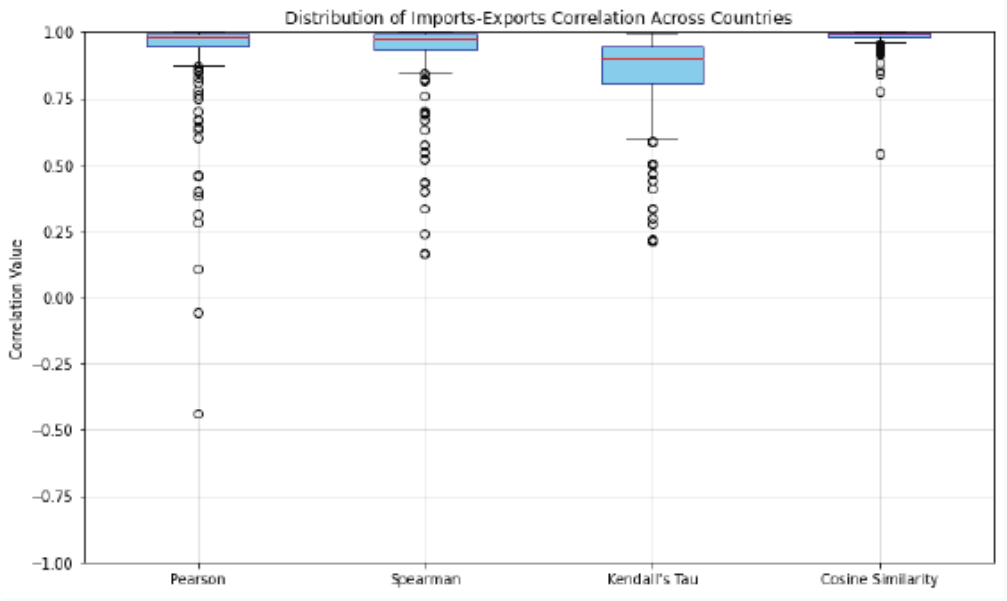
The import-export correlation

(a) Country–year imports–exports correlation (1960–2022)



Note: Goods and services data retrieved from World Bank. Unbalanced panel. Best–fit line weighted by total trade.

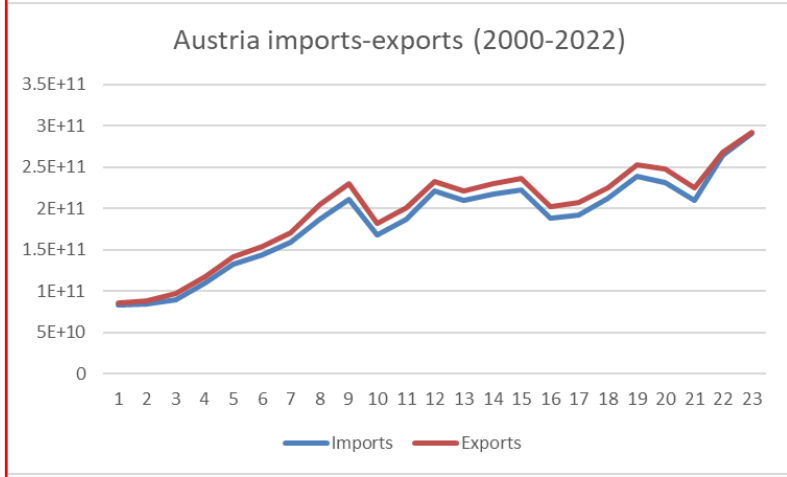
(b) Longitudinal country–level correlation (1960–2022))

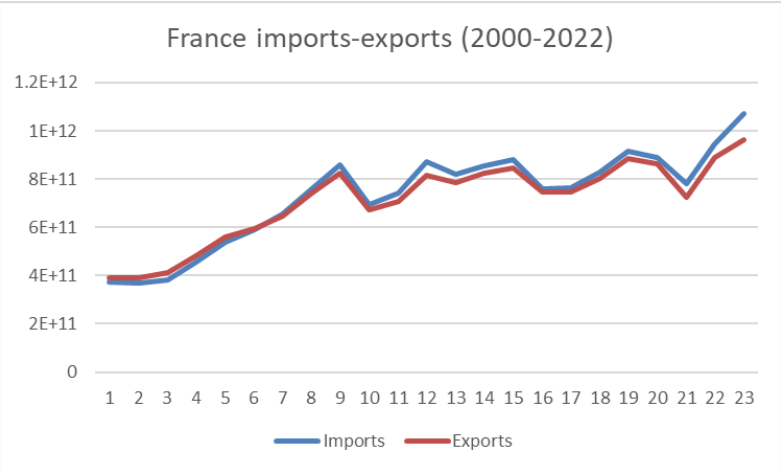


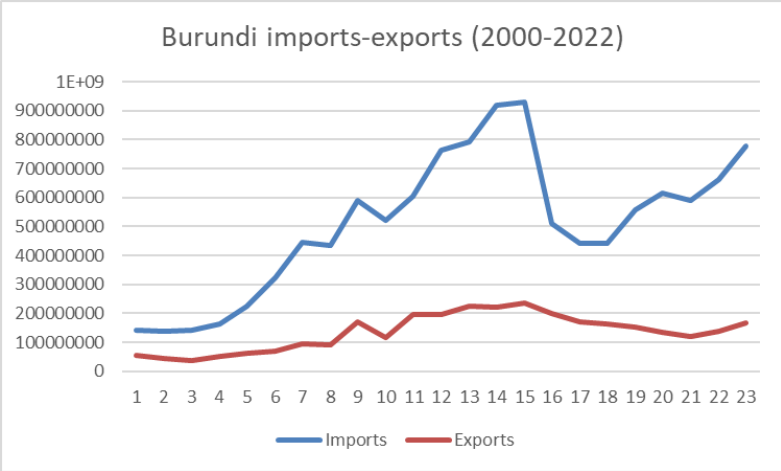
Note: Goods and services data retrieved from World Bank. Unbalanced panel. Several correlation metrics used.

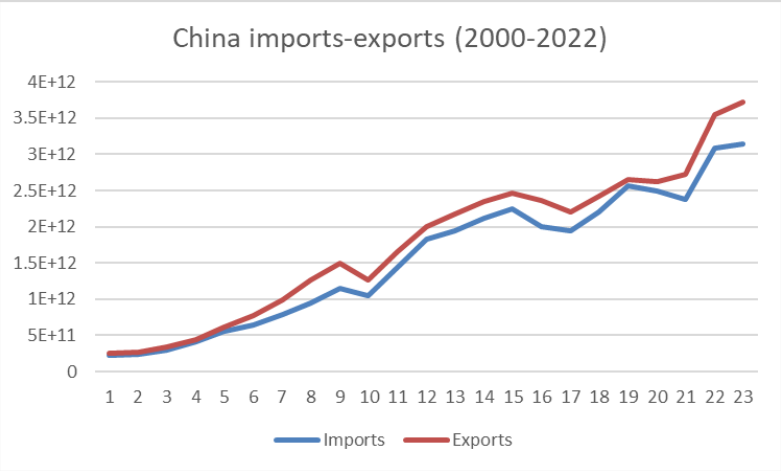
- **Bebczuk (2008):** "[...] a puzzling yet strong correlation exists between imports and exports".
- **Los and Verspagen (2002):** countries cannot sustain increasing and permanent current account deficits, unless capital inflows keep coming in infinitely and increasing amounts – which usually they cannot.

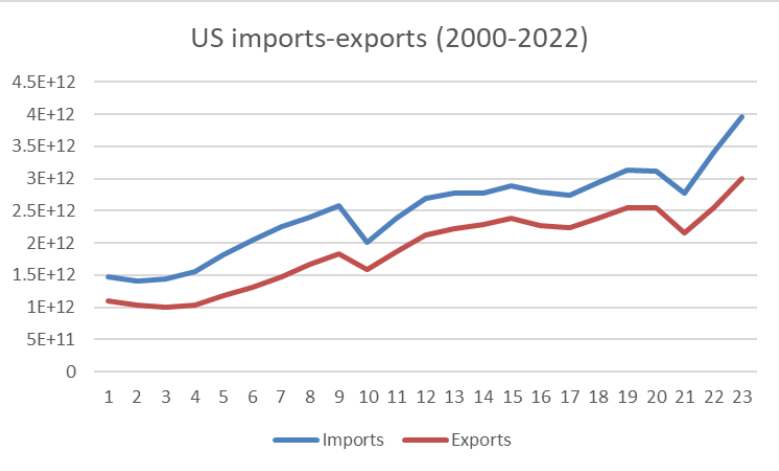
The import-export correlation



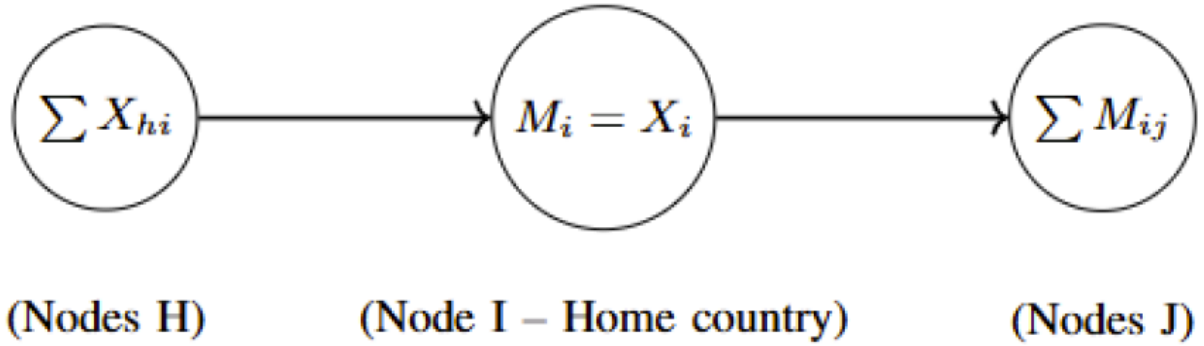








From the correlation, to the law



Due to high import-export correlation, we assume for simplicity:

$$(1) \quad \sum_{k=1}^{n} X_{ki} = \sum_{j=1}^{m} M_{ij}$$

We allow the network to grow by taking differentials:

$$(2) \quad \sum_{k=1}^{n} \frac{dX_{hi}}{X_{hi}} = \sum_{j=1}^{m} \frac{dM_{ij}}{M_{ij}}$$

Exports as imports, and foreign-imports in weighted averages:

$$(3) \quad \widehat{M_i} = \widehat{M_{ij}}$$

Applying the income elasticity of demand for imports identity:

$$(4) \quad \widehat{Y_i} \cdot \theta_i = \widehat{Y_j} \cdot \theta_j$$

In a closed network income elasticity of import demand is the income elasticity of export demand of someone else:

$$(5) \quad \widehat{Y_i} = \widehat{Y_j} \cdot \frac{\varepsilon_i}{\theta_i}$$

Thirlwall's law as a network eigenproblem

$$\widehat{Y_1} = \widehat{Y_k} \cdot \frac{\varepsilon_1}{\theta_1}$$

$$\widehat{Y_2} = \widehat{Y_k} \cdot \frac{\varepsilon_2}{\theta_2}$$

$$\widehat{Y_3} = \widehat{Y_k} \cdot \frac{\varepsilon_3}{\theta_3}$$

$$\vdots$$

$$\widehat{Y_n} = \widehat{Y_k} \cdot \frac{\varepsilon_n}{\theta_n}$$

Import-share matrix:

$$W = \begin{pmatrix} 0 & \dots & w_{1,n} \\ \vdots & \ddots & \vdots \\ w_{n,1} & \dots & 0 \end{pmatrix}, \qquad \sum_{j=1}^{n} w_{ij} = 1$$

Retrieving import elasticities from weighted (random) export elasticities of trade partners:

$$\theta_i = \sum_{j=1}^{n} w_{ij} \cdot \varepsilon_j$$

Income elasticity of export/import demand ratio matrix:

$$D = \begin{pmatrix} \frac{\varepsilon_i}{\theta_i} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{\varepsilon_n}{\theta_n} \end{pmatrix}$$

With matrix *A = DW, solving* through the power-method to retrieve the (eigen)vector of growth rates:

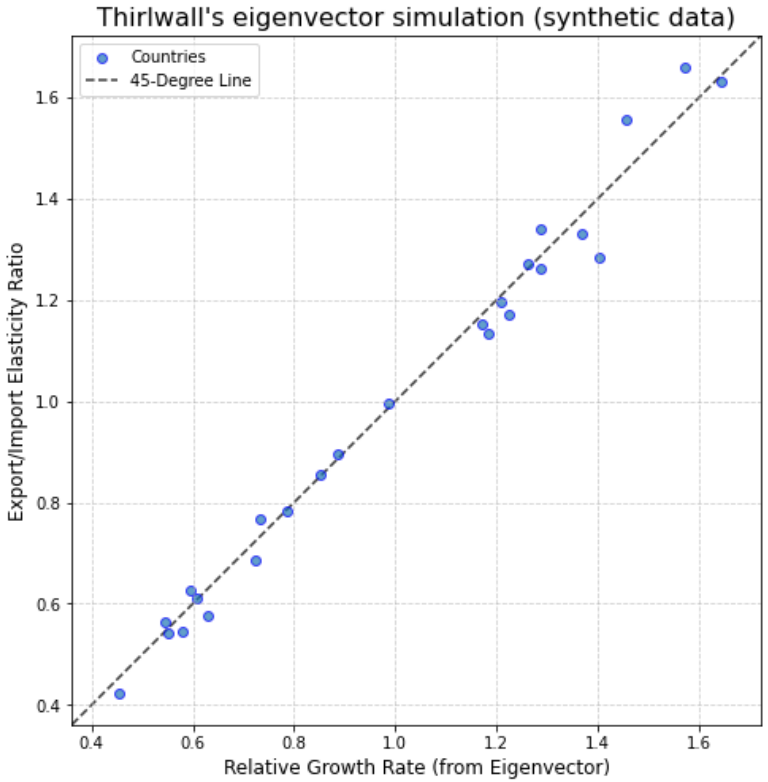
$$\hat{Y} = A\hat{Y} \;\rightarrow\; (I - A)\hat{Y} = 0$$

$$\boxed{\hat{Y} = \left(\widehat{Y_1}, \dots, \widehat{Y_n}\right)}$$

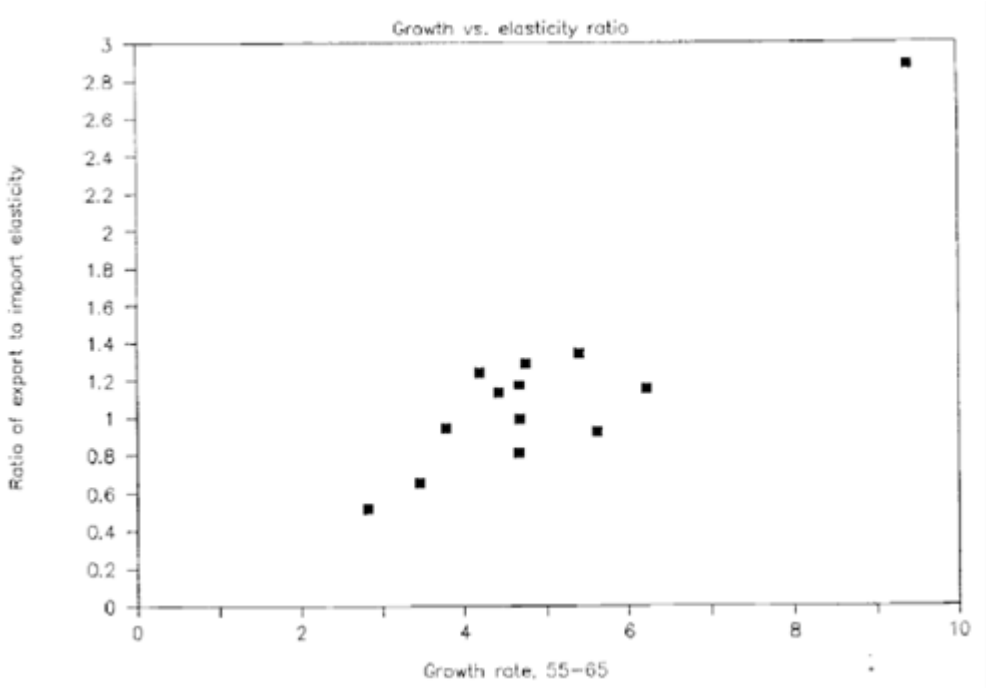
Relative growth rates compatible with the composition of imports and exports

Thirlwall's eigenvector vs Krugman's 45-degree rule

(a) Thirlwall–eigenvector simulation using synthetic data



(b) Krugman's (1988) 45-degree rule using empirical data



Fredholm Alternative (work in progress)

$$\begin{matrix} X_i = M_i + e_i \\ \vdots \\ X_n = M_n + e_n \end{matrix} \quad , \sum_{i=1}^{n} e_i = 0$$

The eigenvector applies anyways

Discussion

- Thirlwall’s eigenvector is the global foreign trade multiplier. It explains:
  - Output synchronization (growth eigen-path)
  - Center-periphery dynamics
  - Provides mathematical formalization to dependency theory

- It suggests demand is non-neutral in the long run.

- It suggests growth is a global phenomena that is ‘’distributed’’ through income elasticities of exports and imports to each country. Therefore, income elasticities are structural.

- Bilateral trade treaties won't generate a long-term increase in relative growth rates, unless they increase the income elasticity ratio. The converse is true for trade sanctions.

- Countries cannot simultaneously increase their income elasticity ratios (neo-mercantilism).

Thanks! Gracias!