Measuring globalisation and global value chains (GVCs) through Input-Output techniques

PhD Summer School on European Industrial, Innovation and Trade Policy in the Era of Strategic Autonomy

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Outline

1. Why are we here?
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8. Measuring value added trade
9. Measuring value extraction in Global Value Chains (GVCs)
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Why are we here? Cotton and Grapes

Reconsider the famous Ricardian example:
"**England** has a comparative advantage in **cloth** whereas Portugal has a comparative advantage in **wine**"

Pattern of trade (and *gains* derived) unambiguous, **but**:

- who produces the **cotton** to make **cloth**?
- who produces the **grapes** to make **wine**?

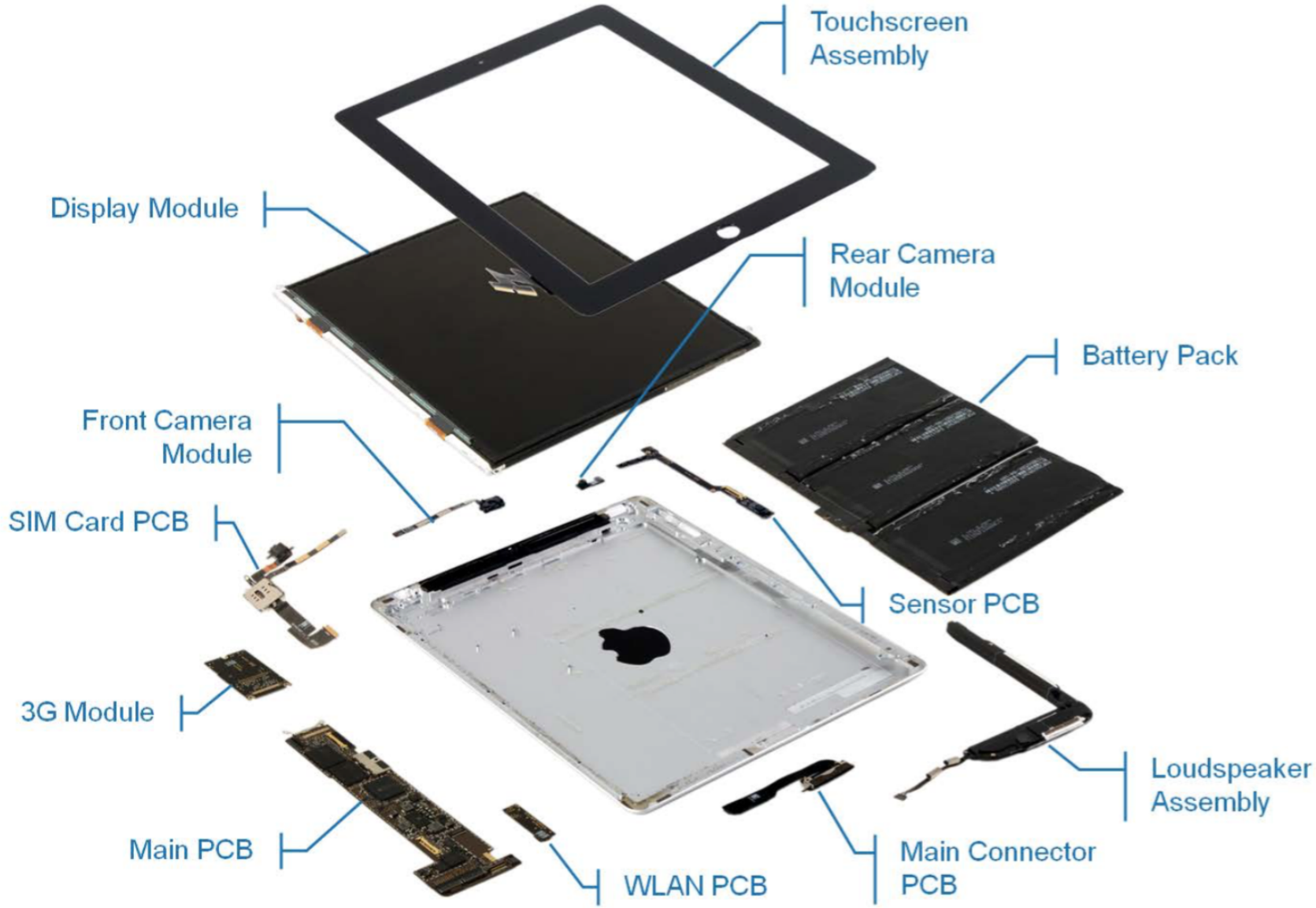
What if...

- **Portugal** has an **advantage** in producing **cotton**, and
- **England** has an **advantage** in producing **grapes**

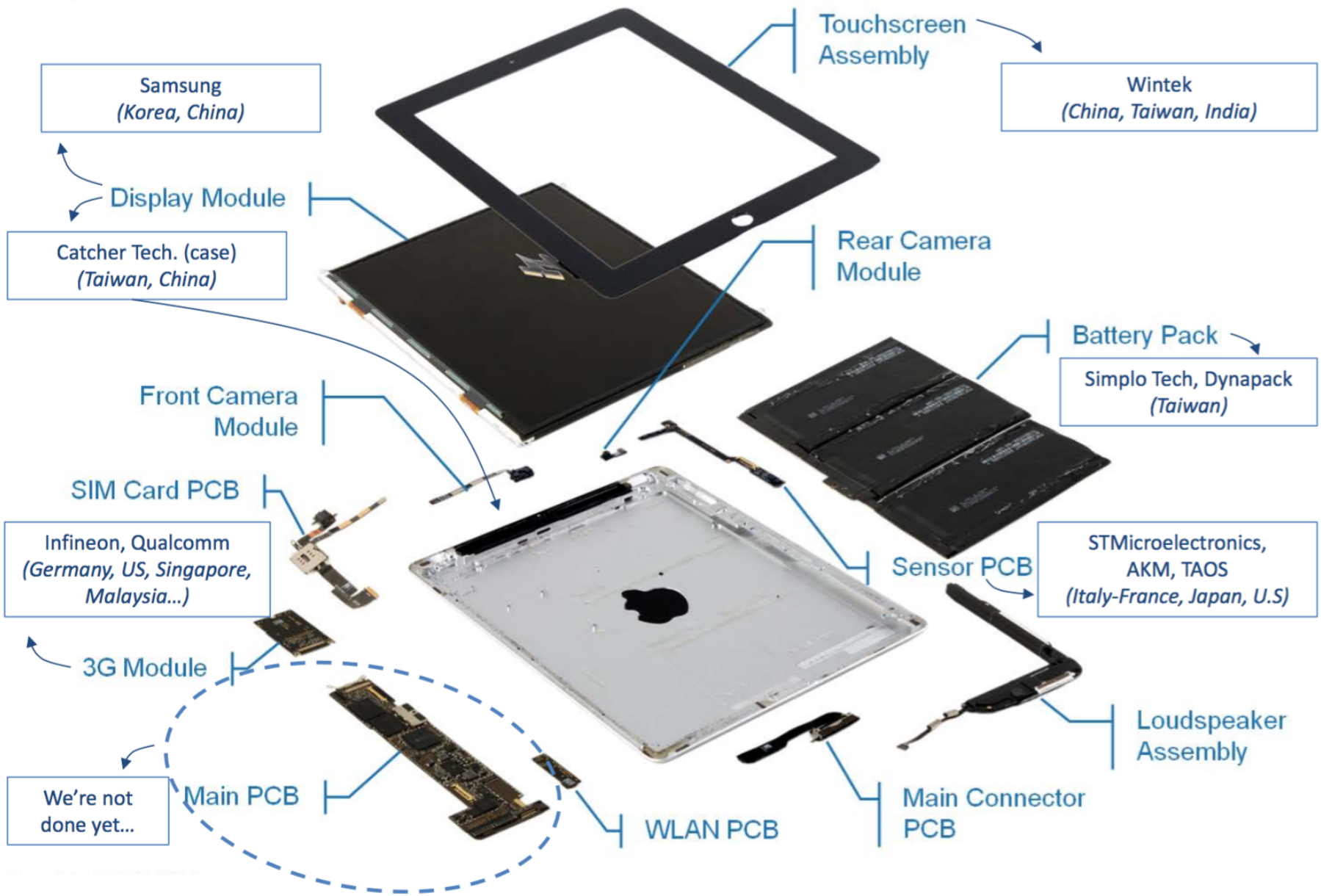
Is the pattern of trade still determined by comparative advantage in **final goods** only?

Comparative advantage of England over Portugal in cloth (relative to wine) **depends on** *absolute advantage* of Portugal over England in cotton used as input by the English cloth industry.

From Cotton and Grapes to. . . Apple's iPad 3



Apple's iPad 3 (cont'd)



Measuring Globalisation and Global Value Chains (GVCs)

- Difficult to give **macroeconomic** dimension to global production and input trade from recollection of **case studies**;
- **Conventional measures** of **trade**: apparent increase in exports/imports may just mirror inputs circulating **without** value being added;
- We need techniques to measure **value added** circulating across countries, **embodied** in inputs until their use as final products.

To study inter-country inter-industry interdependencies we may use Global Input-Output Analysis.

A simplified IRIO global system

Consider an inter-regional input-output (IRIO) production and trade scheme with 3 *endogenous* countries/regions (c, p, r) and two industries (1 y 2):

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		Uses	Intermediate						Final			Total
3	Resources	Country	c	c	p	p	r	r	c	p	r	GO
4	Country	Industry	1	2	1	2	1	2				
5	c	1	Z_{11}^{cc}	Z_{12}^{cc}	Z_{11}^{cp}	Z_{12}^{cp}	Z_{11}^{cr}	Z_{12}^{cr}	F_1^{cc}	F_1^{cp}	F_1^{cr}	Q_1^{c}
6	c	2	Z_{21}^{cc}	Z_{22}^{cc}	Z_{21}^{cp}	Z_{22}^{cp}	Z_{21}^{cr}	Z_{22}^{cr}	F_2^{cc}	F_2^{cp}	F_2^{cr}	Q_2^{c}
7	p	1	Z_{11}^{pc}	Z_{12}^{pc}	Z_{11}^{pp}	Z_{12}^{pp}	Z_{11}^{pr}	Z_{12}^{pr}	F_1^{pc}	F_1^{pp}	F_1^{pr}	Q_1^{p}
8	p	2	Z_{21}^{pc}	Z_{22}^{pc}	Z_{21}^{pp}	Z_{22}^{pp}	Z_{21}^{pr}	Z_{22}^{pr}	F_2^{pc}	F_2^{pp}	F_2^{pr}	Q_2^{p}
9	r	1	Z_{11}^{rc}	Z_{12}^{rc}	Z_{11}^{rp}	Z_{12}^{rp}	Z_{11}^{rr}	Z_{12}^{rr}	F_1^{rc}	F_1^{rp}	F_1^{rr}	Q_1^{r}
10	r	2	Z_{21}^{rc}	Z_{22}^{rc}	Z_{21}^{rp}	Z_{22}^{rp}	Z_{21}^{rr}	Z_{22}^{rr}	F_2^{rc}	F_2^{rp}	F_2^{rr}	Q_2^{r}
11	GVA		Y_1^{c}	Y_2^{c}	Y_1^{p}	Y_2^{p}	Y_1^{r}	Y_2^{r}				
12	GO		Q_1^{c}	Q_2^{c}	Q_1^{p}	Q_2^{p}	Q_1^{r}	Q_2^{r}				

Figura 1: Simplified IRIO system (endogenous RoW)

What does each component mean?

		Uses	Intermediate						Final			Total
Resources	Country	c	c	p	p	r	r	c	p	r	GO
Country	Industry	1	2	1	2	1	2				
c	1	Z_{11}^{cc}	Z_{12}^{cc}	Z_{11}^{cp}	Z_{12}^{cp}	Z_{11}^{cr}	Z_{12}^{cr}	F_1^{cc}	F_1^{cp}	F_1^{cr}	Q_1^{c}
c	2	Z_{21}^{cc}	Z_{22}^{cc}	Z_{21}^{cp}	Z_{22}^{cp}	Z_{21}^{cr}	Z_{22}^{cr}	F_2^{cc}	F_2^{cp}	F_2^{cr}	Q_2^{c}
p	1	Z_{11}^{pc}	Z_{12}^{pc}	Z_{11}^{pp}	Z_{12}^{pp}	Z_{11}^{pr}	Z_{12}^{pr}	F_1^{pc}	F_1^{pp}	F_1^{pr}	Q_1^{p}
p	2	Z_{21}^{pc}	Z_{22}^{pc}	Z_{21}^{pp}	Z_{22}^{pp}	Z_{21}^{pr}	Z_{22}^{pr}	F_2^{pc}	F_2^{pp}	F_2^{pr}	Q_2^{p}
r	1	Z_{11}^{rc}	Z_{12}^{rc}	Z_{11}^{rp}	Z_{12}^{rp}	Z_{11}^{rr}	Z_{12}^{rr}	F_1^{rc}	F_1^{rp}	F_1^{rr}	Q_1^{r}
r	2	Z_{21}^{rc}	Z_{22}^{rc}	Z_{21}^{rp}	Z_{22}^{rp}	Z_{21}^{rr}	Z_{22}^{rr}	F_2^{rc}	F_2^{rp}	F_2^{rr}	Q_2^{r}
GVA		Y_1^{c}	Y_2^{c}	Y_1^{p}	Y_2^{p}	Y_1^{r}	Y_2^{r}			domestic	
GO		Q_1^{c}	Q_2^{c}	Q_1^{p}	Q_2^{p}	Q_1^{r}	Q_2^{r}			imported/exported	
										value added (and other comp.)	
										gross output	

31	Z_{12}^{cc}	intermediate use of products from industry 1 in country C by industry 2 in country C
32	Z_{21}^{pc}	intermediate use of products from industry 2 in country P by industry 1 of country C
33	F_2^{pp}	use of products from industry 2 in country P for consumption/investment in country P
34	Y_1^{p}	gross value added (plus net taxes on products) of industry 1 in country P
35	Q_2^{r}	gross output of industry 2 in country R

Figura 2: IRIO system components

A macroscopic view of the world economy

	AQ	AR	AS	AT	AU	AV	AW	AX	AY	AZ	BA	BB	BC	BD	BE	BF	BG	BH	BI	BJ
1	Global Input-Output Table																			
2	*(aggregated by macro areas in 10^9 current USD)*																			
3			Intermediates (Z)					Fixed Capital (K)					Consumption & other final uses (C)							
4	yr	from/to	Z_US	Z_EU	Z_RoGN	Z_CHN	Z_RoGS	K_US	K_EU	K_RoGN	K_CHN	K_RoGS	C_US	C_EU	C_RoGN	C_CHN	C_RoGS	q		
5	2019	US	**14760.1**	301.0	394.1	99.9	471.5	**4017.0**	72.8	78.1	22.2	103.7	**15932.1**	91.7	185.3	62.4	326.6	36918.7		
6	2019	EU	233.4	**13066.0**	520.0	201.4	653.7	74.0	**2981.4**	122.4	53.7	197.4	240.2	**10193.0**	320.6	56.5	449.7	29363.6		
7	2019	RoGN	410.4	498.7	**10764.6**	454.5	572.7	121.6	99.6	**2874.9**	66.4	155.7	255.3	174.8	**9511.4**	97.3	291.9	26349.9		
8	2019	CHN	168.6	197.9	286.5	**20263.8**	611.2	78.1	57.1	85.7	**5719.7**	193.9	168.7	113.1	168.3	**7335.6**	278.2	35726.3		
9	2019	RoGS	499.6	745.8	642.8	856.3	**19075.9**	134.3	64.7	70.8	59.6	**4202.4**	402.2	291.2	261.5	159.4	**14919.6**	42386.2		
10	2019	y	20846.6	14554.2	13741.8	13850.4	21001.3													
11	2019	q	36918.7	29363.6	26349.9	35726.3	42386.2													
12																				
13	Domestically produced components:																			
14			Intermediates (Z)					Fixed Capital (K)					Consumption & other final uses (C)							
15	yr	from/to	Z_US	Z_EU	Z_RoGN	Z_CHN	Z_RoGS	K_US	K_EU	K_RoGN	K_CHN	K_RoGS	C_US	C_EU	C_RoGN	C_CHN	C_RoGS			
16	2019	US	14760.1	0.0	0.0	0.0	0.0	4017.0	0.0	0.0	0.0	0.0	15932.1	0.0	0.0	0.0	0.0			
17	2019	EU	0.0	11121.3	0.0	0.0	0.0	0.0	2600.1	0.0	0.0	0.0	0.0	9368.0	0.0	0.0	0.0			
18	2019	RoGN	0.0	0.0	10465.2	0.0	0.0	0.0	0.0	2821.2	0.0	0.0	0.0	0.0	9399.0	0.0	0.0			
19	2019	CHN	0.0	0.0	0.0	20263.8	0.0	0.0	0.0	0.0	5719.7	0.0	0.0	0.0	0.0	7335.6	0.0			
20	2019	RoGS	0.0	0.0	0.0	0.0	17708.9	0.0	0.0	0.0	0.0	4025.4	0.0	0.0	0.0	0.0	14222.7			
21																				
22	World Trade Matrices:																			
23			Intermediates (Z)						Fixed Capital (K)						Consumption & other final uses (C)					
24	yr	from/to	Z_US	Z_EU	Z_RoGN	Z_CHN	Z_RoGS	ex	K_US	K_EU	K_RoGN	K_CHN	K_RoGS	ex	C_US	C_EU	C_RoGN	C_CHN	C_RoGS	ex
25	2019	US	0.0	301.0	394.1	99.9	471.5	1266.5	0.0	72.8	78.1	22.2	103.7	276.9	0.0	91.7	185.3	62.4	326.6	666.1
26	2019	EU	233.4	1944.8	520.0	201.4	653.7	3553.4	74.0	381.3	122.4	53.7	197.4	828.8	240.2	825.0	320.6	56.5	449.7	1892.0
27	2019	RoGN	410.4	498.7	299.4	454.5	572.7	2235.6	121.6	99.6	53.7	66.4	155.7	497.1	255.3	174.8	112.4	97.3	291.9	931.8
28	2019	CHN	168.6	197.9	286.5	0.0	611.2	1264.2	78.1	57.1	85.7	0.0	193.9	414.8	168.7	113.1	168.3	0.0	278.2	728.3
29	2019	RoGS	499.6	745.8	642.8	856.3	1367.0	4111.5	134.3	64.7	70.8	59.6	177.0	506.4	402.2	291.2	261.5	159.4	697.0	1811.3
30		m	1312.1	3688.1	2142.8	1612.2	3676.0	m	408.0	675.5	410.7	201.9	827.8	m	1066.5	1495.8	1048.2	375.6	2043.4	
31		BoT	-45.5	-134.8	92.8	-348.0	435.5	BoT	-131.1	153.3	86.4	212.9	-321.4	BoT	-400.4	396.2	-116.4	352.7	-232.1	
32																				
33	*Source: Author's computation based on OECD-ICIO (2023 Edition)*																			

Structural proportions of the world economy

Structural proportions of the global economy:

Global Income (%)

	1995	2007	2019
US	0.251	0.254	0.248
EU	0.260	0.246	0.173
RoGN	0.293	0.213	0.164
CHN	0.024	0.061	0.165
RoGS	0.172	0.226	0.250
Total	1.000	1.000	1.000

Global Consumption (%)

	1995	2007	2019
US	0.263	0.276	0.273
EU	0.263	0.246	0.174
RoGN	0.274	0.212	0.168
CHN	0.021	0.042	0.124
RoGS	0.179	0.224	0.261
Total	1.000	1.000	1.000

Global Investment (%)

	1995	2007	2019
US	0.223	0.237	0.204
EU	0.236	0.239	0.151
RoGN	0.343	0.210	0.149
CHN	0.033	0.096	0.273
RoGS	0.164	0.219	0.224
Total	1.000	1.000	1.000

BoT - Intermediates (Z) in 10^9 USD

	1995	2007	2019
US	-47.9	-331.4	-45.5
EU	-21.5	-278.7	-134.8
RoGN	26.1	52.8	92.8
CHN	-22.8	-74.5	-348.0
RoGS	66.0	631.8	435.5
Total	0.0	0.0	0.0

BoT - Consumption (C) in 10^9 USD

	1995	2007	2019
US	-10.8	-290.1	-400.4
EU	71.8	247.3	396.2
RoGN	-24.2	-27.4	-116.4
CHN	31.5	278.6	352.7
RoGS	-68.3	-208.5	-232.1
Total	0.0	0.0	0.0

BoT - Investment (K) in 10^9 USD

	1995	2007	2019
US	-25.1	-91.3	-131.1
EU	53.7	145.8	153.3
RoGN	65.1	81.1	86.4
CHN	0.0	112.3	212.9
RoGS	-93.7	-247.9	-321.4
Total	0.0	0.0	0.0

Source: Author's computation based on OECD-ICIO (2023 Edition)

Matrix notation and operations

$$\underset{(2\times 1)}{\boldsymbol{v}} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad \underset{(2\times 2)}{\widehat{\boldsymbol{v}}} = \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}, \quad \underset{(2\times 2)}{\boldsymbol{B}} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$\underset{(1\times 2)}{\boldsymbol{v}^T} = \begin{bmatrix} v_1 & v_2 \end{bmatrix}, \quad \underset{(2\times 2)}{\boldsymbol{B}^T} = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix}$$

$$\underset{(1\times 2)}{\boldsymbol{v}^T\boldsymbol{B}} = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} v_1 b_{11} + v_2 b_{21} & v_1 b_{12} + v_2 b_{22} \end{bmatrix}$$

$$\underset{(2\times 1)}{\boldsymbol{B}\boldsymbol{f}} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} b_{11} f_1 + b_{12} f_2 \\ b_{21} f_1 + b_{22} f_2 \end{bmatrix}$$

Matrix notation and operations (cont'd)

$$\underset{(2\times 2)}{\widehat{\boldsymbol{v}}\boldsymbol{B}} = \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} v_1 b_{11} & v_1 b_{12} \\ v_2 b_{21} & v_2 b_{22} \end{bmatrix}$$

$$\underset{(2\times 2)}{\boldsymbol{B}\widehat{\boldsymbol{f}}} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix} = \begin{bmatrix} b_{11} f_1 & b_{12} f_2 \\ b_{21} f_1 & b_{22} f_2 \end{bmatrix}$$

$$\underset{(1\times 2)}{\mathbf{1}^T\boldsymbol{B}} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} b_{11} + b_{21} & b_{12} + b_{22} \end{bmatrix}$$

$$\underset{(2\times 1)}{\boldsymbol{B}\mathbf{1}} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} b_{11} + b_{12} \\ b_{21} + b_{22} \end{bmatrix}$$

Operationalising the accounting scheme

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		Uses	Intermediate						Final			Tot
3	Res	Country	c	c	p	p	r	r	c	p	r	GO
4	Country	Industry	1	2	1	2	1	2				
5	c	1	Z_{11}^{cc}	Z_{12}^{cc}	Z_{11}^{cp}	Z_{12}^{cp}	Z_{11}^{cr}	Z_{12}^{cr}	F_1^{cc}	F_1^{cp}	F_1^{cr}	Q_1^{c}
6	c	2	Z_{21}^{cc}	Z_{22}^{cc}	Z_{21}^{cp}	Z_{22}^{cp}	Z_{21}^{cr}	Z_{22}^{cr}	F_2^{cc}	F_2^{cp}	F_2^{cr}	Q_2^{c}
7	p	1	Z_{11}^{pc}	Z_{12}^{pc}	Z_{11}^{pp}	Z_{12}^{pp}	Z_{11}^{pr}	Z_{12}^{pr}	F_1^{pc}	F_1^{pp}	F_1^{pr}	Q_1^{p}
8	p	2	Z_{21}^{pc}	Z_{22}^{pc}	Z_{21}^{pp}	Z_{22}^{pp}	Z_{21}^{pr}	Z_{22}^{pr}	F_2^{pc}	F_2^{pp}	F_2^{pr}	Q_2^{p}
9	r	1	Z_{11}^{rc}	Z_{12}^{rc}	Z_{11}^{rp}	Z_{12}^{rp}	Z_{11}^{rr}	Z_{12}^{rr}	F_1^{rc}	F_1^{rp}	F_1^{rr}	Q_1^{r}
10	r	2	Z_{21}^{rc}	Z_{22}^{rc}	Z_{21}^{rp}	Z_{22}^{rp}	Z_{21}^{rr}	Z_{22}^{rr}	F_2^{rc}	F_2^{rp}	F_2^{rr}	Q_2^{r}
11	GVA		Y_1^{c}	Y_2^{c}	Y_1^{p}	Y_2^{p}	Y_1^{r}	Y_2^{r}	0	0	0	0
12	GO		Q_1^{c}	Q_2^{c}	Q_1^{p}	Q_2^{p}	Q_1^{r}	Q_2^{r}	0	0	0	0

	N	O	P	Q	R	S	T	U	V	W	X
2	Uses	Intermediate						Final			Tot
3	Res	c	c	p	p	r	r	c	p	r	GO
5	c	$\mathbf{Z}_{cc}$		$\mathbf{Z}_{cp}$		$\mathbf{Z}_{cr}$		$\mathbf{f}_{cc}$	$\mathbf{f}_{cp}$	$\mathbf{f}_{cr}$	$\mathbf{q}_c$
7	p	$\mathbf{Z}_{pc}$		$\mathbf{Z}_{pp}$		$\mathbf{Z}_{pr}$		$\mathbf{f}_{pc}$	$\mathbf{f}_{pp}$	$\mathbf{f}_{pr}$	$\mathbf{q}_p$
9	r	$\mathbf{Z}_{rc}$		$\mathbf{Z}_{rp}$		$\mathbf{Z}_{rr}$		$\mathbf{f}_{rc}$	$\mathbf{f}_{rp}$	$\mathbf{f}_{rr}$	$\mathbf{q}_r$
11	GVA	$\mathbf{y}_c^T$		$\mathbf{y}_p^T$		$\mathbf{y}_r^T$		0	0	0	0
12	GO	$\mathbf{q}_c^T$		$\mathbf{q}_p^T$		$\mathbf{q}_r^T$		0	0	0	0

$$\underset{(2\times 2)}{\boldsymbol{Z}_{cp}} = \begin{bmatrix} z_{11}^{cp} & z_{12}^{cp} \\ z_{21}^{cp} & z_{22}^{cp} \end{bmatrix}, \quad \underset{(2\times 1)}{\boldsymbol{f}_{cc}} = \begin{bmatrix} f_1^{cc} \\ f_2^{cc} \end{bmatrix}, \quad \underset{(2\times 1)}{\boldsymbol{q}_c} = \begin{bmatrix} q_1^{c} \\ q_2^{c} \end{bmatrix},$$

$$\underset{(1\times 2)}{\boldsymbol{y}_c^T} = \begin{bmatrix} y_1^{c} & y_2^{c} \end{bmatrix}$$

Operationalising the accounting scheme: two circuits

	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU	AV
1		Row view (expenditure circuit):												Column view (income circuit):									
2	Uses	Intermediate						Final			Tot		Uses	Intermediate						Final			Tot
3	Res	c	c	p	p	r	r	c	p	r	GO		Res	c	c	p	p	r	r	c	p	r	GO
4																							
5	c	Z_{cc}		Z_{cp}		Z_{cr}		f_{cc}	f_{cp}	f_{cr}	q_c		c	Z_{cc}		Z_{cp}		Z_{cr}		f_{cc}	f_{cp}	f_{cr}	q_c
6																							
7	p	Z_{pc}		Z_{pp}		Z_{pr}		f_{pc}	f_{pp}	f_{pr}	q_p		p	Z_{pc}		Z_{pp}		Z_{pr}		f_{pc}	f_{pp}	f_{pr}	q_p
8																							
9	r	Z_{rc}		Z_{rp}		Z_{rr}		f_{rc}	f_{rp}	f_{rr}	q_r		r	Z_{rc}		Z_{rp}		Z_{rr}		f_{rc}	f_{rp}	f_{rr}	q_r
10																							
11	GVA	y_c^T		y_p^T		y_r^T		0	0	0	0		GVA	y_c^T		y_p^T		y_r^T		0	0	0	0
12	GO	q_c^T		q_p^T		q_r^T		0	0	0	0		GO	q_c^T		q_p^T		q_r^T		0	0	0	0

- Expenditure circuit: uses of output (domestic intermediate, domestic exported, domestic final, exported final)
- Income circuit: costs of output (domestic inputs, imported inputs, value added and net taxes)

Expenditure circuit

Algebraic formulation:

$$\begin{bmatrix} \boldsymbol{Z}_{cc} & \boldsymbol{Z}_{cp} & \boldsymbol{Z}_{cr} \\ \boldsymbol{Z}_{pc} & \boldsymbol{Z}_{pp} & \boldsymbol{Z}_{pr} \\ \boldsymbol{Z}_{rc} & \boldsymbol{Z}_{rp} & \boldsymbol{Z}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} & \boldsymbol{f}_{cr} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} & \boldsymbol{f}_{pr} \\ \boldsymbol{f}_{rc} & \boldsymbol{f}_{rp} & \boldsymbol{f}_{rr} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \\ \boldsymbol{q}_r \end{bmatrix}$$

For country C:

$$\boldsymbol{q}_c = \boldsymbol{Z}_{cc}\mathbf{1} + \boldsymbol{Z}_{cp}\mathbf{1} + \boldsymbol{Z}_{cr}\mathbf{1} + \boldsymbol{f}_{cc} + \boldsymbol{f}_{cp} + \boldsymbol{f}_{cr}$$

For country P:

$$\boldsymbol{q}_p = \boldsymbol{Z}_{pc}\mathbf{1} + \boldsymbol{Z}_{pp}\mathbf{1} + \boldsymbol{Z}_{pr}\mathbf{1} + \boldsymbol{f}_{pc} + \boldsymbol{f}_{pp} + \boldsymbol{f}_{pr}$$

For country R:

$$\boldsymbol{q}_r = \boldsymbol{Z}_{rc}\mathbf{1} + \boldsymbol{Z}_{rp}\mathbf{1} + \boldsymbol{Z}_{rr}\mathbf{1} + \boldsymbol{f}_{rc} + \boldsymbol{f}_{rp} + \boldsymbol{f}_{rr}$$

	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ
1		Row view (expenditure circuit):									
2	Uses	Intermediate						Final			Tot
3	Res	c	c	p	p	r	r	c	p	r	GO
4											
5	c	$\boldsymbol{Z}_{cc}$		$\boldsymbol{Z}_{cp}$		$\boldsymbol{Z}_{cr}$		$\boldsymbol{f}_{cc}$	$\boldsymbol{f}_{cp}$	$\boldsymbol{f}_{cr}$	$\boldsymbol{q}_c$
6											
7	p	$\boldsymbol{Z}_{pc}$		$\boldsymbol{Z}_{pp}$		$\boldsymbol{Z}_{pr}$		$\boldsymbol{f}_{pc}$	$\boldsymbol{f}_{pp}$	$\boldsymbol{f}_{pr}$	$\boldsymbol{q}_p$
8											
9	r	$\boldsymbol{Z}_{rc}$		$\boldsymbol{Z}_{rp}$		$\boldsymbol{Z}_{rr}$		$\boldsymbol{f}_{rc}$	$\boldsymbol{f}_{rp}$	$\boldsymbol{f}_{rr}$	$\boldsymbol{q}_r$
10											
11	GVA	$\boldsymbol{y}_c^T$		$\boldsymbol{y}_p^T$		$\boldsymbol{y}_r^T$		0	0	0	0
12	GO	$\boldsymbol{q}_c^T$		$\boldsymbol{q}_p^T$		$\boldsymbol{q}_r^T$		0	0	0	0

Income circuit

Algebraic formulation:

$$\begin{bmatrix} \mathbf{1}^T & \mathbf{1}^T & \mathbf{1}^T \end{bmatrix} \begin{bmatrix} \mathbf{Z}_{cc} & \mathbf{Z}_{cp} & \mathbf{Z}_{cr} \\ \mathbf{Z}_{pc} & \mathbf{Z}_{pp} & \mathbf{Z}_{pr} \\ \mathbf{Z}_{rc} & \mathbf{Z}_{rp} & \mathbf{Z}_{rr} \end{bmatrix} + \begin{bmatrix} \mathbf{y}_c^T & \mathbf{y}_p^T & \mathbf{y}_r^T \end{bmatrix} = \begin{bmatrix} \mathbf{q}_c^T & \mathbf{q}_p^T & \mathbf{q}_r^T \end{bmatrix}$$

For country C:

$$\mathbf{q}_c^T = \mathbf{1}^T \mathbf{Z}_{cc} + \mathbf{1}^T \mathbf{Z}_{pc} + \mathbf{1}^T \mathbf{Z}_{rc} + \mathbf{y}_c^T$$

For country P:

$$\mathbf{q}_p^T = \mathbf{1}^T \mathbf{Z}_{cp} + \mathbf{1}^T \mathbf{Z}_{pp} + \mathbf{1}^T \mathbf{Z}_{rp} + \mathbf{y}_p^T$$

For country R:

$$\mathbf{q}_r^T = \mathbf{1}^T \mathbf{Z}_{cr} + \mathbf{1}^T \mathbf{Z}_{pr} + \mathbf{1}^T \mathbf{Z}_{rr} + \mathbf{y}_r^T$$

	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU	AV
1		Column view (income circuit):									
2	Uses	Intermediate						Final			Tot
3	Res	c	c	p	p	r	r	c	p	r	GO
4											
5	c	$\mathbf{Z}_{cc}$		$\mathbf{Z}_{cp}$		$\mathbf{Z}_{cr}$		$\mathbf{f}_{cc}$	$\mathbf{f}_{cp}$	$\mathbf{f}_{cr}$	$\mathbf{q}_c$
6											
7	p	$\mathbf{Z}_{pc}$		$\mathbf{Z}_{pp}$		$\mathbf{Z}_{pr}$		$\mathbf{f}_{pc}$	$\mathbf{f}_{pp}$	$\mathbf{f}_{pr}$	$\mathbf{q}_p$
8											
9	r	$\mathbf{Z}_{rc}$		$\mathbf{Z}_{rp}$		$\mathbf{Z}_{rr}$		$\mathbf{f}_{rc}$	$\mathbf{f}_{rp}$	$\mathbf{f}_{rr}$	$\mathbf{q}_r$
10											
11	GVA	$\mathbf{y}_c^T$		$\mathbf{y}_p^T$		$\mathbf{y}_r^T$		0	0	0	0
12	GO	$\mathbf{q}_c^T$		$\mathbf{q}_p^T$		$\mathbf{q}_r^T$		0	0	0	0

Interface with macro accounting

From the perspective of country C:

Panel (A)

Uses	Intermediate						Final			Tot
Res	c	c	p	p	r	r	c	p	r	GO
c	$\mathbf{Z}_{cc}$		$\mathbf{Z}_{cp}$		$\mathbf{Z}_{cr}$		$\mathbf{f}_{cc}$	$\mathbf{f}_{cp}$	$\mathbf{f}_{cr}$	$\mathbf{q}_c$
p	$\mathbf{Z}_{pc}$		$\mathbf{Z}_{pp}$		$\mathbf{Z}_{pr}$		$\mathbf{f}_{pc}$	$\mathbf{f}_{pp}$	$\mathbf{f}_{pr}$	$\mathbf{q}_p$
r	$\mathbf{Z}_{rc}$		$\mathbf{Z}_{rp}$		$\mathbf{Z}_{rr}$		$\mathbf{f}_{rc}$	$\mathbf{f}_{rp}$	$\mathbf{f}_{rr}$	$\mathbf{q}_r$
GVA	$\mathbf{y}_c^T$		$\mathbf{y}_p^T$		$\mathbf{y}_r^T$		0	0	0	0
GO	$\mathbf{q}_c^T$		$\mathbf{q}_p^T$		$\mathbf{q}_r^T$		0	0	0	0

Panel (B)

Uses	Intermediate						Final			Tot
Res	c	c	p	p	r	r	c	p	r	GO
c	$\mathbf{Z}_{cc}$		$\mathbf{Z}_{cp}$		$\mathbf{Z}_{cr}$		$\mathbf{f}_{cc}$	$\mathbf{f}_{cp}$	$\mathbf{f}_{cr}$	$\mathbf{q}_c$
p	$\mathbf{Z}_{pc}$		$\mathbf{Z}_{pp}$		$\mathbf{Z}_{pr}$		$\mathbf{f}_{pc}$	$\mathbf{f}_{pp}$	$\mathbf{f}_{pr}$	$\mathbf{q}_p$
r	$\mathbf{Z}_{rc}$		$\mathbf{Z}_{rp}$		$\mathbf{Z}_{rr}$		$\mathbf{f}_{rc}$	$\mathbf{f}_{rp}$	$\mathbf{f}_{rr}$	$\mathbf{q}_r$
GVA	$\mathbf{y}_c^T$		$\mathbf{y}_p^T$		$\mathbf{y}_r^T$		0	0	0	0
GO	$\mathbf{q}_c^T$		$\mathbf{q}_p^T$		$\mathbf{q}_r^T$		0	0	0	0

Panel (C)

Uses	Intermediate						Final			Tot
Res	c	c	p	p	r	r	c	p	r	GO
c	$\mathbf{Z}_{cc}$		$\mathbf{Z}_{cp}$		$\mathbf{Z}_{cr}$		$\mathbf{f}_{cc}$	$\mathbf{f}_{cp}$	$\mathbf{f}_{cr}$	$\mathbf{q}_c$
p	$\mathbf{Z}_{pc}$		$\mathbf{Z}_{pp}$		$\mathbf{Z}_{pr}$		$\mathbf{f}_{pc}$	$\mathbf{f}_{pp}$	$\mathbf{f}_{pr}$	$\mathbf{q}_p$
r	$\mathbf{Z}_{rc}$		$\mathbf{Z}_{rp}$		$\mathbf{Z}_{rr}$		$\mathbf{f}_{rc}$	$\mathbf{f}_{rp}$	$\mathbf{f}_{rr}$	$\mathbf{q}_r$
GVA	$\mathbf{y}_c^T$		$\mathbf{y}_p^T$		$\mathbf{y}_r^T$		0	0	0	0
GO	$\mathbf{q}_c^T$		$\mathbf{q}_p^T$		$\mathbf{q}_r^T$		0	0	0	0

Exports (by product)
Imports (by product)
Domestic final demand nationally produced

Domestic final demand (by product)
Intermediate imports (by industry)

Net product (by product)
GVA (by industry)
GO (by industry and product)
Intermediate consumption

Figura 3: IRIO system: interface with national accounts

Interface with macro accounting (cont'd)

From the perspective of country C:

$$\text{Exports}_c = \boldsymbol{Z}_{cp}\mathbf{1} + \boldsymbol{Z}_{cr}\mathbf{1} + \boldsymbol{f}_{cp} + \boldsymbol{f}_{cr}$$

$$\underset{\text{(by product)}}{\text{Imports}_c} = \boldsymbol{Z}_{pc}\mathbf{1} + \boldsymbol{Z}_{rc}\mathbf{1} + \boldsymbol{f}_{pc} + \boldsymbol{f}_{rc}$$

$$\underset{\text{(by industry)}}{\text{Intermediate Imports}_c} = \mathbf{1}^T\boldsymbol{Z}_{pc} + \mathbf{1}^T\boldsymbol{Z}_{rc}$$

$$\text{Domestic final demand}_c = \boldsymbol{f}_{cc} + \boldsymbol{f}_{pc} + \boldsymbol{f}_{rc}$$

$$\underset{\text{(nationally produced)}}{\text{Domestic final demand}_c} = \boldsymbol{f}_{cc}$$

$$\text{Net Product}_c = \boldsymbol{f}_{cc} + \boldsymbol{f}_{cp} + \boldsymbol{f}_{cr} + \boldsymbol{Z}_{cp}\mathbf{1} + \boldsymbol{Z}_{cr}\mathbf{1}$$

$$\text{Intermediate Consumption}_c = \mathbf{1}^T\boldsymbol{Z}_{cc} + \mathbf{1}^T\boldsymbol{Z}_{pc} + \mathbf{1}^T\boldsymbol{Z}_{rc}$$

GDP of country C derived from the IRIO scheme:

$$\text{GDP}_c = \underbrace{(\boldsymbol{f}_{cc} + \boldsymbol{f}_{pc} + \boldsymbol{f}_{rc})}_{\text{(domestic absorption)}} + \underbrace{(\boldsymbol{Z}_{cp}\mathbf{1} + \boldsymbol{Z}_{cr}\mathbf{1} + \boldsymbol{f}_{cp} + \boldsymbol{f}_{cr})}_{\text{(exports)}} - \underbrace{(\boldsymbol{Z}_{pc}\mathbf{1} + \boldsymbol{Z}_{rc}\mathbf{1} + \boldsymbol{f}_{pc} + \boldsymbol{f}_{rc})}_{\text{(imports)}}$$

$$\underbrace{\mathbf{1}^T\text{GDP}_c}_{\text{(GDP expenditure side)}} = \underbrace{\boldsymbol{y}_c^T\mathbf{1}}_{\text{(GVA income side)}}$$

Interface with *global* macro accounting

	A	B	C	D	E	F	G	H	I	J	K
1											
2	Uses	Intermediate						Final			Tot
3	Res	c	c	p	p	r	r	c	p	r	GO
4											
5	c	Z_{cc}		Z_{cp}		Z_{cr}		f_{cc}	f_{cp}	f_{cr}	q_c
6											
7	p	Z_{pc}		Z_{pp}		Z_{pr}		f_{pc}	f_{pp}	f_{pr}	q_p
8											
9	r	Z_{rc}		Z_{rp}		Z_{rr}		f_{rc}	f_{rp}	f_{rr}	q_r
10											
11	GVA	y_c^T		y_p^T		y_r^T		0	0	0	0
12	GO	q_c^T		q_p^T		q_r^T		0	0	0	0
13											
14		GO (by industry and product)									
15		GVA (by industry)									
16		Final demand (by product)									
17		Intermediate consumption									

$$\begin{aligned} \text{GVA}^T &= \boldsymbol{y}_c^T + \boldsymbol{y}_p^T + \boldsymbol{y}_r^T \\ \text{Final Demand} &= (\boldsymbol{f}_{cc} + \boldsymbol{f}_{pc} + \boldsymbol{f}_{rc}) + \\ &\quad + (\boldsymbol{f}_{cp} + \boldsymbol{f}_{pp} + \boldsymbol{f}_{rp}) + \\ &\quad + (\boldsymbol{f}_{cr} + \boldsymbol{f}_{pr} + \boldsymbol{f}_{rr}) \end{aligned}$$

Global accounting identity:

$$\text{GVA}^T \mathbf{1} = \mathbf{1}^T \text{Final Demand} (= \text{GDP})$$

Global GVA is equal to global final demand (at the *aggregate*)

Global Sourcing Matrix

(scheme with $n = 2$ countries y $k = 2$ industries in each country)

$$\underset{(n\times k)\times(n\times k)}{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}_{cc} & \boldsymbol{A}_{cp} \\ \boldsymbol{A}_{pc} & \boldsymbol{A}_{pp} \end{bmatrix} := \begin{bmatrix} \boldsymbol{Z}_{cc} & \boldsymbol{Z}_{cp} \\ \boldsymbol{Z}_{pc} & \boldsymbol{Z}_{pp} \end{bmatrix} \begin{bmatrix} \widehat{\boldsymbol{q}}_c^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \widehat{\boldsymbol{q}}_p^{-1} \end{bmatrix}$$

$$\begin{bmatrix} a_{11}^{cc} & a_{12}^{cc} & a_{11}^{cp} & a_{12}^{cp} \\ a_{21}^{cc} & a_{22}^{cc} & a_{21}^{cp} & a_{22}^{cp} \\ a_{11}^{pc} & a_{12}^{pc} & a_{11}^{pp} & a_{12}^{pp} \\ a_{21}^{pc} & a_{22}^{pc} & a_{21}^{pp} & a_{22}^{pp} \end{bmatrix} = \begin{bmatrix} z_{11}^{cc}/q_1^c & z_{12}^{cc}/q_2^c & z_{11}^{cp}/q_1^p & z_{12}^{cp}/q_2^p \\ z_{21}^{cc}/q_1^c & z_{22}^{cc}/q_2^c & z_{21}^{cp}/q_1^p & z_{22}^{cp}/q_2^p \\ z_{11}^{pc}/q_1^c & z_{12}^{pc}/q_2^c & z_{11}^{pp}/q_1^p & z_{12}^{pp}/q_2^p \\ z_{21}^{pc}/q_1^c & z_{22}^{pc}/q_2^c & z_{21}^{pp}/q_1^p & z_{22}^{pp}/q_2^p \end{bmatrix}$$

Element of matrix $\boldsymbol{A}$, for example, $a_{12}^{cp} = \dfrac{z_{12}^{cp}}{q_2^p}$:

"inputs from industry 1 in country C required by industry 2 in country P, per unit of gross output of industry 2 in country P"

Expenditure circuit in *intensive* terms

Using $\boldsymbol{A}$, matrix $\boldsymbol{Z}$ may be expressed as:

$$\begin{bmatrix} \boldsymbol{Z}_{cc} & \boldsymbol{Z}_{cp} \\ \boldsymbol{Z}_{pc} & \boldsymbol{Z}_{pp} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{cc} & \boldsymbol{A}_{cp} \\ \boldsymbol{A}_{pc} & \boldsymbol{A}_{pp} \end{bmatrix} \begin{bmatrix} \widehat{\boldsymbol{q}}_c & \mathbf{0} \\ \mathbf{0} & \widehat{\boldsymbol{q}}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{cc}\widehat{\boldsymbol{q}}_c & \boldsymbol{A}_{cp}\widehat{\boldsymbol{q}}_p \\ \boldsymbol{A}_{pc}\widehat{\boldsymbol{q}}_c & \boldsymbol{A}_{pp}\widehat{\boldsymbol{q}}_p \end{bmatrix}$$

So that the expenditure circuit:

$$\begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}_{cc} & \boldsymbol{Z}_{cp} \\ \boldsymbol{Z}_{pc} & \boldsymbol{Z}_{pp} \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

may be written as:

$$\begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{cc} & \boldsymbol{A}_{cp} \\ \boldsymbol{A}_{pc} & \boldsymbol{A}_{pp} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Expenditure circuit in *intensive* terms

Circularity:

$$\begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{cc} & \boldsymbol{A}_{cp} \\ \boldsymbol{A}_{pc} & \boldsymbol{A}_{pp} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

From each country's perspective:

$$\boldsymbol{q}_c = \boldsymbol{A}_{cc}\boldsymbol{q}_c + \boldsymbol{A}_{cp}\boldsymbol{q}_p + \boldsymbol{f}_{cc} + \boldsymbol{f}_{cp}$$

$$\boldsymbol{q}_p = \underset{(1)}{\boldsymbol{A}_{pc}\boldsymbol{q}_c} + \underset{(2)}{\boldsymbol{A}_{pp}\boldsymbol{q}_p} + \underset{(3)}{\boldsymbol{f}_{pc}} + \underset{(4)}{\boldsymbol{f}_{pp}}$$

From the perspective of country P:

(1) foreign gross output activates input production at home;

(2) domestic gross output activates input production at home;

(3) Foreign consumption/investment activates home production of final products (e.g., machine exported to country C);

(4) Domestic consumption/investment activates home production of final products (e.g., clothing demanded by consumers in country P).

Solution to the expenditure system

Start from the expenditure system and note that $\boldsymbol{I}\boldsymbol{q} = \boldsymbol{q}$:

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{cc} & \boldsymbol{A}_{cp} \\ \boldsymbol{A}_{pc} & \boldsymbol{A}_{pp} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

reordering terms:

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} - \begin{bmatrix} \boldsymbol{A}_{cc} & \boldsymbol{A}_{cp} \\ \boldsymbol{A}_{pc} & \boldsymbol{A}_{pp} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

reordering terms:

$$\begin{bmatrix} \boldsymbol{I} - \boldsymbol{A}_{cc} & -\boldsymbol{A}_{cp} \\ -\boldsymbol{A}_{pc} & \boldsymbol{I} - \boldsymbol{A}_{pp} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

If matrix $(\boldsymbol{I} - \boldsymbol{A})$ admits an inverse:

$$\begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} - \boldsymbol{A}_{cc} & -\boldsymbol{A}_{cp} \\ -\boldsymbol{A}_{pc} & \boldsymbol{I} - \boldsymbol{A}_{pp} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \square$$

Global Leontief Matrix

(scheme with $n = 2$ countries y $k = 2$ industries in each country)

$$\underset{(n\times k)\times(n\times k)}{(\boldsymbol{I}-\boldsymbol{A})}^{-1} = \begin{bmatrix} \boldsymbol{I}-\boldsymbol{A}_{cc} & -\boldsymbol{A}_{cp} \\ -\boldsymbol{A}_{pc} & \boldsymbol{I}-\boldsymbol{A}_{pp} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{B}_{cc} & \boldsymbol{B}_{cp} \\ \boldsymbol{B}_{pc} & \boldsymbol{B}_{pp} \end{bmatrix} = \boldsymbol{B}$$

$$\boldsymbol{B} = \begin{bmatrix} 1-a_{11}^{cc} & -a_{12}^{cc} & -a_{11}^{cp} & -a_{12}^{cp} \\ -a_{21}^{cc} & 1-a_{22}^{cc} & -a_{21}^{cp} & a_{22}^{cp} \\ -a_{11}^{pc} & -a_{12}^{pc} & 1-a_{11}^{pp} & -a_{12}^{pp} \\ -a_{21}^{pc} & -a_{22}^{pc} & -a_{21}^{pp} & 1-a_{22}^{pp} \end{bmatrix}^{-1} = \begin{bmatrix} b_{11}^{cc} & b_{12}^{cc} & b_{11}^{cp} & b_{12}^{cp} \\ b_{21}^{cc} & b_{22}^{cc} & b_{21}^{cp} & b_{22}^{cp} \\ b_{11}^{pc} & b_{12}^{pc} & b_{11}^{pp} & b_{12}^{pp} \\ b_{21}^{pc} & b_{22}^{pc} & b_{21}^{pp} & b_{22}^{pp} \end{bmatrix}$$

Element of matrix $\boldsymbol{B}$, for example, b_{12}^{cp}:

"direct and indirect inputs from industry 1 in country C required by industry 2 in country P to produce a unit of *final* output of industry 2 in country P"

Global interdependencies

Solution to the expenditure system:

$$\begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_{cc} & \boldsymbol{B}_{cp} \\ \boldsymbol{B}_{pc} & \boldsymbol{B}_{pp} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

operating with matrices:

$$\begin{bmatrix} \boldsymbol{q}_c \\ \boldsymbol{q}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_{cc}\boldsymbol{f}_{cc} + \boldsymbol{B}_{cp}\boldsymbol{f}_{pc} & \boldsymbol{B}_{cc}\boldsymbol{f}_{cp} + \boldsymbol{B}_{cp}\boldsymbol{f}_{pp} \\ \boldsymbol{B}_{pc}\boldsymbol{f}_{cc} + \boldsymbol{B}_{pp}\boldsymbol{f}_{pc} & \boldsymbol{B}_{pc}\boldsymbol{f}_{cp} + \boldsymbol{B}_{pp}\boldsymbol{f}_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(direct and indirect) productive interdependencies between countries:

$$\boldsymbol{q}_c = \boldsymbol{B}_{cc}\boldsymbol{f}_{cc} + \boldsymbol{B}_{cp}\boldsymbol{f}_{pc} + \boldsymbol{B}_{cc}\boldsymbol{f}_{cp} + \boldsymbol{B}_{cp}\boldsymbol{f}_{pp}$$
$$\boldsymbol{q}_p = \boldsymbol{B}_{pc}\boldsymbol{f}_{cc} + \boldsymbol{B}_{pp}\boldsymbol{f}_{pc} + \boldsymbol{B}_{pc}\boldsymbol{f}_{cp} + \boldsymbol{B}_{pp}\boldsymbol{f}_{pp}$$

Measuring vertical specialisation

- Second globalisation unbundling since 1990s (Baldwin, 2016): geographical fragmentation of production into 'stages and loops';
- *Offshoring*: share of imported inputs in manufacturing (Feenstra and Hanson, 1999);
- Vertical specialisation: import content of exports (Hummels et al., 2001);
- But "imports often include domestic value added that was exported and then re-imported" (Ahmad et al., 2017, p. 25)

e.g. think of the value added of the fertilizers *produced in England* used by Portugal to produce the cotton sold to England to produce the cloth.

Measuring vertical specialisation: VAX matrix

VAX matrix: value added *content of* gross exports

$$\mathtt{VAX} = \underbrace{\begin{bmatrix} \widehat{\boldsymbol{v}}_c & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widehat{\boldsymbol{v}}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \widehat{\boldsymbol{v}}_r \end{bmatrix}}_{(=\widehat{\boldsymbol{v}})} \underbrace{\begin{bmatrix} \boldsymbol{B}_{cc} & \boldsymbol{B}_{cp} & \boldsymbol{B}_{cr} \\ \boldsymbol{B}_{pc} & \boldsymbol{B}_{pp} & \boldsymbol{B}_{pr} \\ \boldsymbol{B}_{rc} & \boldsymbol{B}_{rp} & \boldsymbol{B}_{rr} \end{bmatrix}}_{(=\boldsymbol{B})} \underbrace{\begin{bmatrix} \boldsymbol{e}_c & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{e}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{e}_r \end{bmatrix}}_{(=\boldsymbol{E})}$$

$$\underset{(n\cdot k)\times n}{\mathtt{VAX}} = \underset{(n\cdot k)\times n}{\widehat{\boldsymbol{v}}\boldsymbol{B}\boldsymbol{E}} = \begin{bmatrix} \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rr}\boldsymbol{e}_r \end{bmatrix}$$

where each region's exports are given by:

$$\begin{aligned} \boldsymbol{e}_c &= \boldsymbol{Z}_{cp}\mathbf{1} + \boldsymbol{Z}_{cr}\mathbf{1} + \boldsymbol{f}_{cp} + \boldsymbol{f}_{cr} \\ \boldsymbol{e}_p &= \boldsymbol{Z}_{pc}\mathbf{1} + \boldsymbol{Z}_{pr}\mathbf{1} + \boldsymbol{f}_{pc} + \boldsymbol{f}_{pr} \\ \boldsymbol{e}_r &= \boldsymbol{Z}_{rc}\mathbf{1} + \boldsymbol{Z}_{rp}\mathbf{1} + \boldsymbol{f}_{rc} + \boldsymbol{f}_{rp} \end{aligned}$$

Measuring vertical specialisation: VAX matrix (cont'd)

Using matrix VAX:

$$\underset{(n\cdot k)\times n}{\mathtt{VAX}} = \widehat{\boldsymbol{v}}\boldsymbol{B}\boldsymbol{E} = \begin{bmatrix} \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rr}\boldsymbol{e}_r \end{bmatrix}$$

we can measure vertical specialisation, taking into account inter-country feedback loops.

From the perspective of country C:

$$\begin{aligned} \mathtt{DVAX1\_c} &= \mathbf{1}^T\widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cc}\boldsymbol{e}_c \\ \mathtt{DVAX2\_c} &= \mathbf{1}^T(\widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cp}\boldsymbol{e}_p + \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cr}\boldsymbol{e}_r) \\ \mathtt{FVAX\_c} &= \mathbf{1}^T(\widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\boldsymbol{e}_c + \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rc}\boldsymbol{e}_c) \end{aligned}$$

Domestic value added content of *own* gross exports (DVAX1_c)

My exports require *domestic* inputs which incorporate *local* value added

$$\underset{(n\cdot k)\times n}{\mathtt{VAX}} = \underset{(n\cdot k)\times n}{\widehat{\boldsymbol{v}}\boldsymbol{B}\boldsymbol{E}} = \begin{bmatrix} \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rr}\boldsymbol{e}_r \end{bmatrix}$$

From the perspective of country C:

$$\mathtt{DVAX1\_c} = \mathbf{1}^T\widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cc}\boldsymbol{e}_c$$

Domestic value added content of *foreign* gross exports (DVAX2_c)

Exports from other countries require inputs from my country and, producing those inputs, requires domestic value added

$$\underset{(n\cdot k)\times n}{\mathtt{VAX}} = \widehat{\boldsymbol{v}}\boldsymbol{B}\boldsymbol{E} = \begin{bmatrix} \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rr}\boldsymbol{e}_r \end{bmatrix}$$

From the perspective of country C:

$$\mathtt{DVAX2\_c} = \mathbf{1}^T(\widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cp}\boldsymbol{e}_p + \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cr}\boldsymbol{e}_r)$$

- It is a measure of *forward participation* in GVCs.

Foreign value added content of gross exports (FVAX_c)

My exports require imported inputs which incorporate value added from foreign countries

$$\underset{(n\cdot k)\times n}{\mathtt{VAX}} = \underset{}{\widehat{\boldsymbol{v}}\boldsymbol{B}\boldsymbol{E}} = \begin{bmatrix} \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pr}\boldsymbol{e}_r \\ \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rc}\boldsymbol{e}_c & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rp}\boldsymbol{e}_p & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rr}\boldsymbol{e}_r \end{bmatrix}$$

From the perspective of country C:

$$\mathtt{FVAX\_c} = \mathbf{1}^T(\widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\boldsymbol{e}_c + \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rc}\boldsymbol{e}_c)$$

- It is a measure of *backward participation* in GVCs.

Decomposing the value added content of gross exports

From the perspective of country C:

$$\begin{aligned}
\mathbf{1}^T &= \boldsymbol{v}_c^T \boldsymbol{B}_{cc} + \boldsymbol{v}_p^T \boldsymbol{B}_{pc} + \boldsymbol{v}_r^T \boldsymbol{B}_{rc} && \text{(income circuit)} \\
\mathbf{1}^T \boldsymbol{e}_c &= \boldsymbol{v}_c^T \boldsymbol{B}_{cc} \boldsymbol{e}_c + \boldsymbol{v}_p^T \boldsymbol{B}_{pc} \boldsymbol{e}_c + \boldsymbol{v}_r^T \boldsymbol{B}_{rc} \boldsymbol{e}_c \\
&= (\boldsymbol{v}_c^T \boldsymbol{B}_{cc} \boldsymbol{e}_c) + (\boldsymbol{v}_p^T \boldsymbol{B}_{pc} \boldsymbol{e}_c + \boldsymbol{v}_r^T \boldsymbol{B}_{rc} \boldsymbol{e}_c) \\
\mathbf{1}^T \boldsymbol{e}_c &= \texttt{DVAX1}_c + \texttt{FVAX}_c \quad \square
\end{aligned}$$

- The sum of domestic (`DVAX1_c`) and foreign (`FVAX_c`) value added incorporated in exports must exhaust their total value.

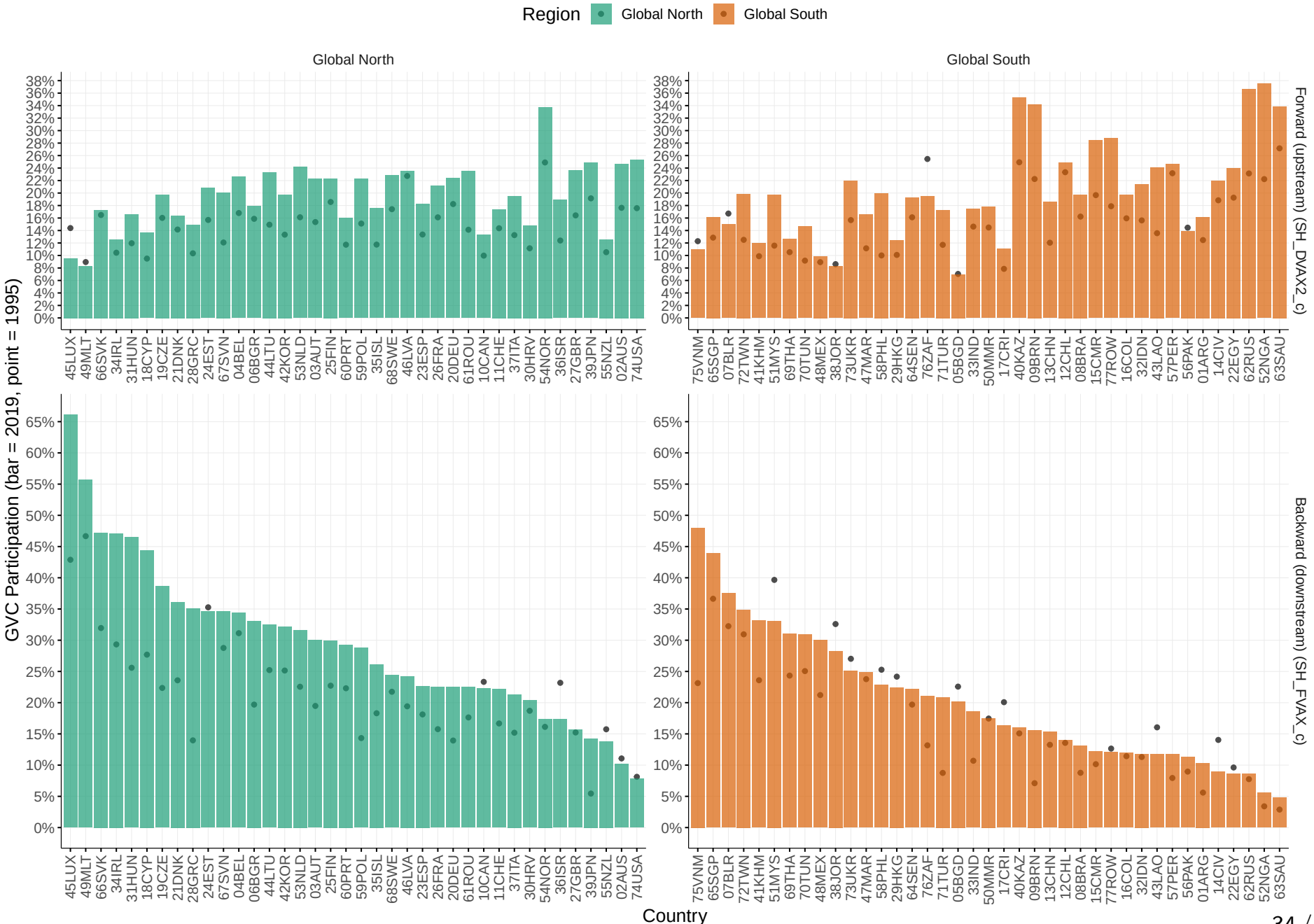
GVC participation index

The Global Value Chain (GVC) participation index is computed as (Ahmad et al., 2017, p. 29):

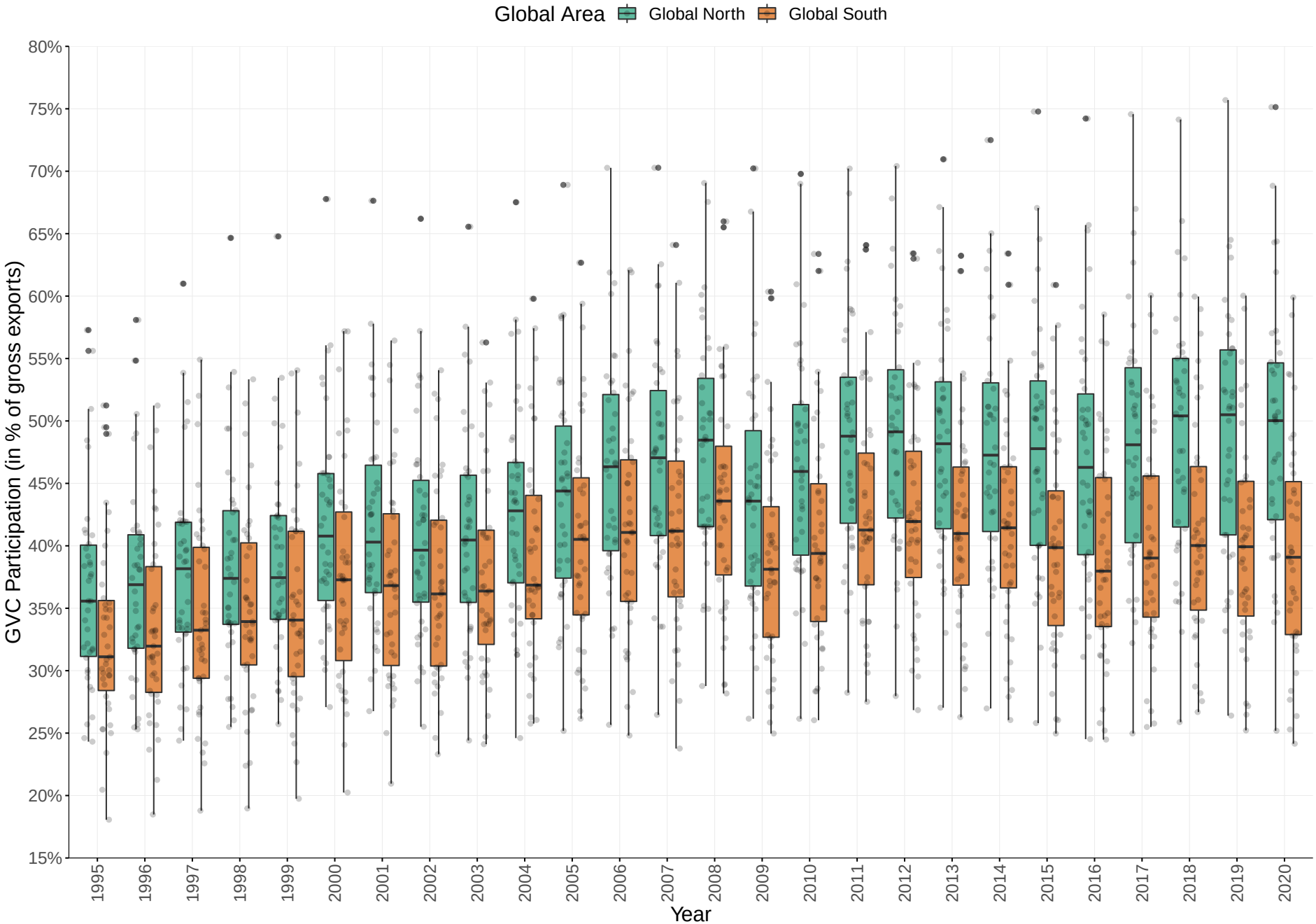
$$\texttt{GVC\_part}_c = \underbrace{\frac{\texttt{DVAX2}_c}{\mathbf{1}^T \boldsymbol{e}_c}}_{\text{(upstream)}} + \underbrace{\frac{\texttt{FVAX}_c}{\mathbf{1}^T \boldsymbol{e}_c}}_{\text{(downstream)}}$$

- Measures the relative importance of a country's exports *upstream* and *downstream*;
- Typically, the bigger the domestic economy (in GDP terms), the smaller the value of the index;
- The *upstream* component captures value added incorporated in inputs which are exported *only* through other countries.

Backward and forward participation in GVCs



GVC participation index



Measuring value added trade

- Value added content of *gross* exports includes *double counting* (Koopman et al., 2014; Los et al., 2016);

- Using an I-O logic, the question should be: what is the value added content of *final* uses?

- Multiple (dis)aggregation possibilities for the concept 'final uses', for instance:

(i) Final demand by geographical source of demand;

(ii) Final output by geographical origin of products;

(iii) Final exports.

We start with (i)...

Measuring value added trade: VAF matrix

VAF matrix: value added *content of* final demand

$$\mathtt{VAF} = \underbrace{\begin{bmatrix} \widehat{v}_c & 0 & 0 \\ 0 & \widehat{v}_p & 0 \\ 0 & 0 & \widehat{v}_r \end{bmatrix}}_{(=\widehat{v})} \underbrace{\begin{bmatrix} B_{cc} & B_{cp} & B_{cr} \\ B_{pc} & B_{pp} & B_{pr} \\ B_{rc} & B_{rp} & B_{rr} \end{bmatrix}}_{(=B)} \underbrace{\begin{bmatrix} f_{cc} & f_{cp} & f_{cr} \\ f_{pc} & f_{pp} & f_{pr} \\ f_{rc} & f_{rp} & f_{rr} \end{bmatrix}}_{(=F)}$$

$$\underset{(n\cdot k)\times n}{\mathtt{VAF}} = \widehat{v}BF =$$

$$\begin{bmatrix} \widehat{v}_c(B_{cc}f_{cc} + B_{cp}f_{pc} + B_{cr}f_{rc}) & \widehat{v}_c(B_{cc}f_{cp} + B_{cp}f_{pp} + B_{cr}f_{rp}) & \widehat{v}_c(B_{cc}f_{cr} + B_{cp}f_{pr} + B_{cr}f_{rr}) \\ \widehat{v}_p(B_{pc}f_{cc} + B_{pp}f_{pc} + B_{pr}f_{rc}) & \widehat{v}_p(B_{pc}f_{cp} + B_{pp}f_{pp} + B_{pr}f_{rp}) & \widehat{v}_p(B_{pc}f_{cr} + B_{pp}f_{pr} + B_{pr}f_{rr}) \\ \widehat{v}_r(B_{rc}f_{cc} + B_{rp}f_{pc} + B_{rr}f_{rc}) & \widehat{v}_r(B_{rc}f_{cp} + B_{rp}f_{pp} + B_{rr}f_{rp}) & \widehat{v}_r(B_{rc}f_{cr} + B_{rp}f_{pr} + B_{rr}f_{rr}) \end{bmatrix}$$

$$\begin{aligned} \mathtt{DVAF}_c = & 1^T(\widehat{v}_c B_{cc} f_{cp} + \widehat{v}_c B_{cp} f_{pp} + \widehat{v}_c B_{cr} f_{rp} + \\ & + \widehat{v}_c B_{cc} f_{cr} + \widehat{v}_c B_{cp} f_{pr} + \widehat{v}_c B_{cr} f_{rr}) \end{aligned}$$

$$\begin{aligned} \mathtt{FVAF}_c = & 1^T(\widehat{v}_p B_{pc} f_{cc} + \widehat{v}_p B_{pp} f_{pc} + \widehat{v}_p B_{pr} f_{rc} + \\ & + \widehat{v}_r B_{rc} f_{cc} + \widehat{v}_r B_{rp} f_{pc} + \widehat{v}_r B_{rr} f_{rc}) \end{aligned}$$

Domestic value added induced by foreign final demand (DVAF_c)

DVAF_c: "total domestic value-added induced in country c by foreign final demand" (Ahmad et al., 2017, p. 30)

$$
\begin{aligned}
\text{DVAF}_c = & \mathbf{1}^T(\widehat{\boldsymbol{v}}_c \boldsymbol{B}_{cc} \boldsymbol{f}_{cp} + \widehat{\boldsymbol{v}}_c \boldsymbol{B}_{cc} \boldsymbol{f}_{cr}) + && (1)\\
& + \mathbf{1}^T(\widehat{\boldsymbol{v}}_c \boldsymbol{B}_{cp} \boldsymbol{f}_{pp} + \widehat{\boldsymbol{v}}_c \boldsymbol{B}_{cr} \boldsymbol{f}_{rr}) + && (2)\\
& + \mathbf{1}^T(\widehat{\boldsymbol{v}}_c \boldsymbol{B}_{cr} \boldsymbol{f}_{rp} + \widehat{\boldsymbol{v}}_c \boldsymbol{B}_{cp} \boldsymbol{f}_{pr}) && (3)
\end{aligned}
$$

(1) Domestic VA of country C incorporated in its own final exports;

(2) Domestic VA incorporated in intermediate exports of country C, used by the importer to produce final products locally consumed;

(3) Domestic VA incorporated in intermediate exports of country C, used by the importer to produce final products for third countries.

Foreign value added induced by domestic final demand (DVAF_c)

FVAF_c: "total foreign value-added embodied in domestic final demand" (Ahmad et al., 2017, p. 30)

$$
\begin{aligned}
\text{FVAF}_c = & \mathbf{1}^T(\widehat{\boldsymbol{v}}_p \boldsymbol{B}_{pc} \boldsymbol{f}_{cc} + \widehat{\boldsymbol{v}}_r \boldsymbol{B}_{rc} \boldsymbol{f}_{cc}) + && (1)\\
& + \mathbf{1}^T(\widehat{\boldsymbol{v}}_p \boldsymbol{B}_{pp} \boldsymbol{f}_{pc} + \widehat{\boldsymbol{v}}_r \boldsymbol{B}_{rr} \boldsymbol{f}_{rc}) + && (2)\\
& + \mathbf{1}^T(\widehat{\boldsymbol{v}}_p \boldsymbol{B}_{pr} \boldsymbol{f}_{rc} + \widehat{\boldsymbol{v}}_r \boldsymbol{B}_{rp} \boldsymbol{f}_{pc}) && (3)
\end{aligned}
$$

(1) Foreign value added incorporated in intermediate imports of country C, to produce final products locally consumed;

(2) Foreign value added coming from a direct trade partner to satisfy final demand of country C;

(3) Foreign value added incorporated in final imports of country C, which has been indirectly transferred through third countries.

Country dependence on foreign demand and inputs

How dependent is country C on foreign demand to generate income?

$$\texttt{SH\_DVAF\_c} = \frac{DVAF_c}{Y_c}$$

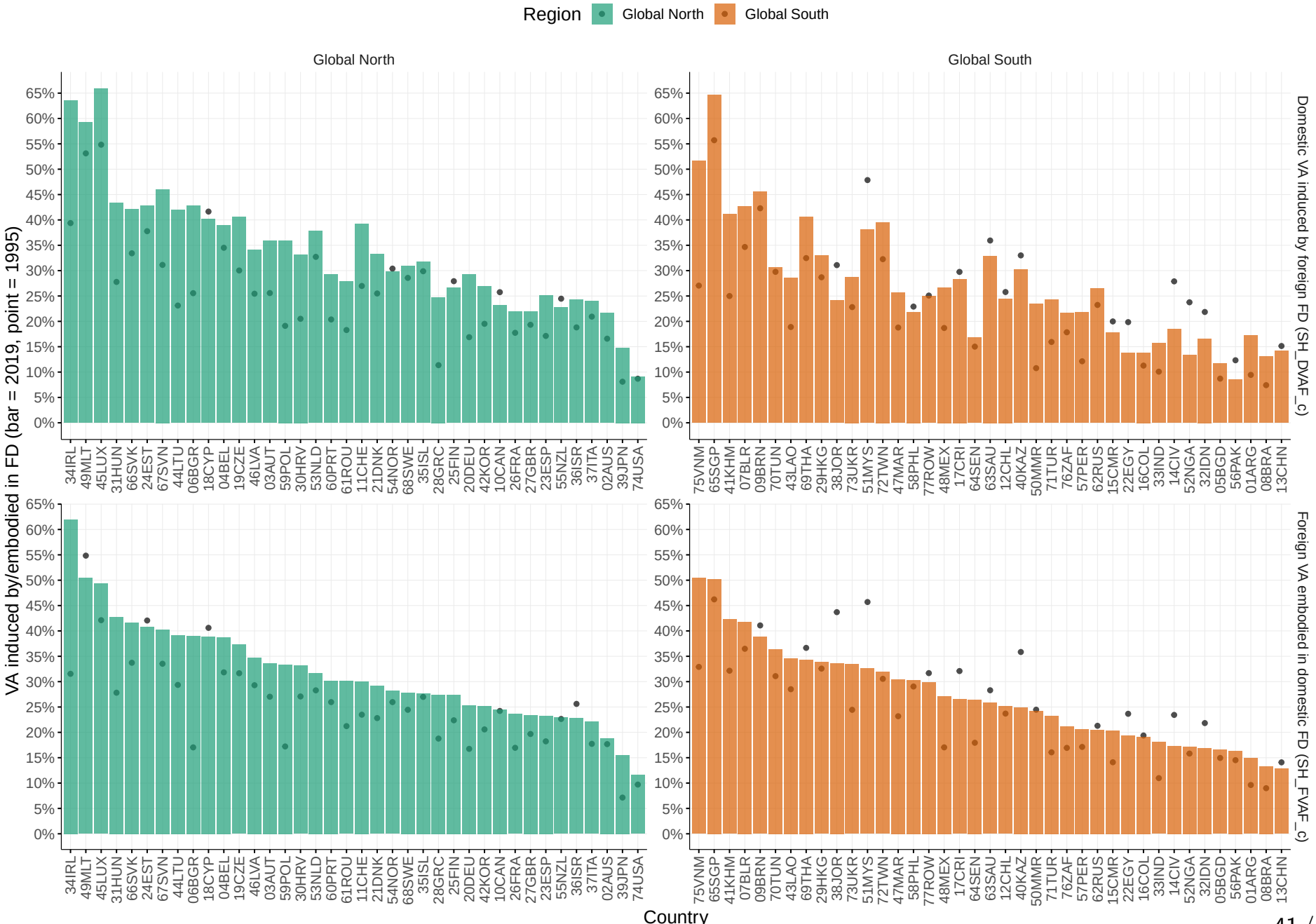
i.e., the share of domestic value added induced by foreign final demand.

How dependent is country C on foreign value added to satisfy domestic final uses (consumption/investment)?

$$\texttt{SH\_FVAF\_c} = \frac{FVAF_c}{FD_c}$$

i.e., the share of foreign value added embodied in domestic final demand.

Country dependence on foreign demand and inputs



Trade Balance in value added terms (TBVAF_c)

- DVAF_c: Domestic value added induced by foreign final demand

  (may be interpreted as total value added *exports*)

- FVAF_c: Foreign value added induced by domestic final demand

  (may be interpreted as total value added *imports*)

Hence, the trade balance in value added terms, TBVAF_c, is:

$$\text{TBVAF}_c = \text{DVAF}_c - \text{FVAF}_c$$

Note that, at the *aggregate* level, for each country C:
(Stehrer, 2012, p. 4)

$$\text{TBVAF}_c = \text{GDP}_c - \text{Domestic Final Demand}_c = \text{X}_c - \text{M}_c = \text{TB}_c$$

Bilateral trade balance matrix

How could we obtain a matrix of *bilateral* trade balances for the world economy?

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	*Global economy (year 2019, values in 10^9 current USD)*															
2	World Trade Matrix: W								World Trade Matrix^T: W^T							
3		Total Trade								Total Trade						
4	from/to	US	EU	RoGN	CHN	RoGS	ex		to/from	US	EU	RoGN	CHN	RoGS	m	
5	US	0.0	465.6	657.6	184.5	901.8	2209.5		US	0.0	547.6	787.3	415.5	1036.1	2786.5	
6	EU	547.6	3151.1	963.0	311.7	1300.9	6274.2		EU	465.6	3151.1	773.0	368.1	1101.7	5859.4	
7	RoGN	787.3	773.0	465.5	618.3	1020.3	3664.5	-	RoGN	657.6	963.0	465.5	540.5	975.1	3601.7	=
8	CHN	415.5	368.1	540.5	0.0	1083.3	2407.3		CHN	184.5	311.7	618.3	0.0	1075.3	2189.7	
9	RoGS	1036.1	1101.7	975.1	1075.3	2241.0	6429.2		RoGS	901.8	1300.9	1020.3	1083.3	2241.0	6547.2	
10	m	2786.5	5859.4	3601.7	2189.7	6547.2			ex	2209.5	6274.2	3664.5	2407.3	6429.2		
11	BoT	-577.0	414.7	62.8	217.6	-118.0			BoT	-577.0	414.7	62.8	217.6	-118.0		

	Q	R	S	T	U	V	W	X
1								
2		BoT = W - W^T						
3			Bilateral Trade Balance					
4		from/to	US	EU	RoGN	CHN	RoGS	BoT
5		US	0.0	-82.0	-129.7	-230.9	-134.3	-577.0
6		EU	82.0	0.0	190.0	-56.4	199.2	414.7
7	=	RoGN	129.7	-190.0	0.0	77.8	45.2	62.8
8		CHN	230.9	56.4	-77.8	0.0	8.0	217.6
9		RoGS	134.3	-199.2	-45.2	-8.0	0.0	-118.0
10		(-BoT)	577.0	-414.7	-62.8	-217.6	118.0	

Bilateral trade balances: gross vs. value added flows

	B	C	D	E	F	G	H
13	*Global economy (year 2019, values in 10^9 current USD)*						
14	TBVAF (Trade Balance in Value Added Flows)						
15	from/to	US	EU	RoGN	CHN	RoGS	*BoT*
16	US	0.0	-120.7	-134.7	-212.5	-109.1	-577.0
17	EU	120.7	0.0	151.7	-35.0	177.3	414.7
18	RoGN	134.7	-151.7	0.0	32.4	47.3	62.8
19	CHN	212.5	35.0	-32.4	0.0	2.5	217.6
20	RoGS	109.1	-177.3	-47.3	-2.5	0.0	-118.0
21							
22	TB (Trade Balance in Gross Flows)						
23	from/to	US	EU	RoGN	CHN	RoGS	*BoT*
24	US	0.0	-82.0	-129.7	-230.9	-134.3	-577.0
25	EU	82.0	0.0	190.0	-56.4	199.2	414.7
26	RoGN	129.7	-190.0	0.0	77.8	45.2	62.8
27	CHN	230.9	56.4	-77.8	0.0	8.0	217.6
28	RoGS	134.3	-199.2	-45.2	-8.0	0.0	-118.0
29							
30	TBVAF - TB (Value Added - Gross Flows)						
31	from/to	US	EU	RoGN	CHN	RoGS	*Diff*
32	US	0.0	-38.7	-5.0	18.4	25.2	0.0
33	EU	38.7	0.0	-38.3	21.4	-21.8	0.0
34	RoGN	5.0	38.3	0.0	-45.4	2.1	0.0
35	CHN	-18.4	-21.4	45.4	0.0	-5.5	0.0
36	RoGS	-25.2	21.8	-2.1	5.5	0.0	0.0

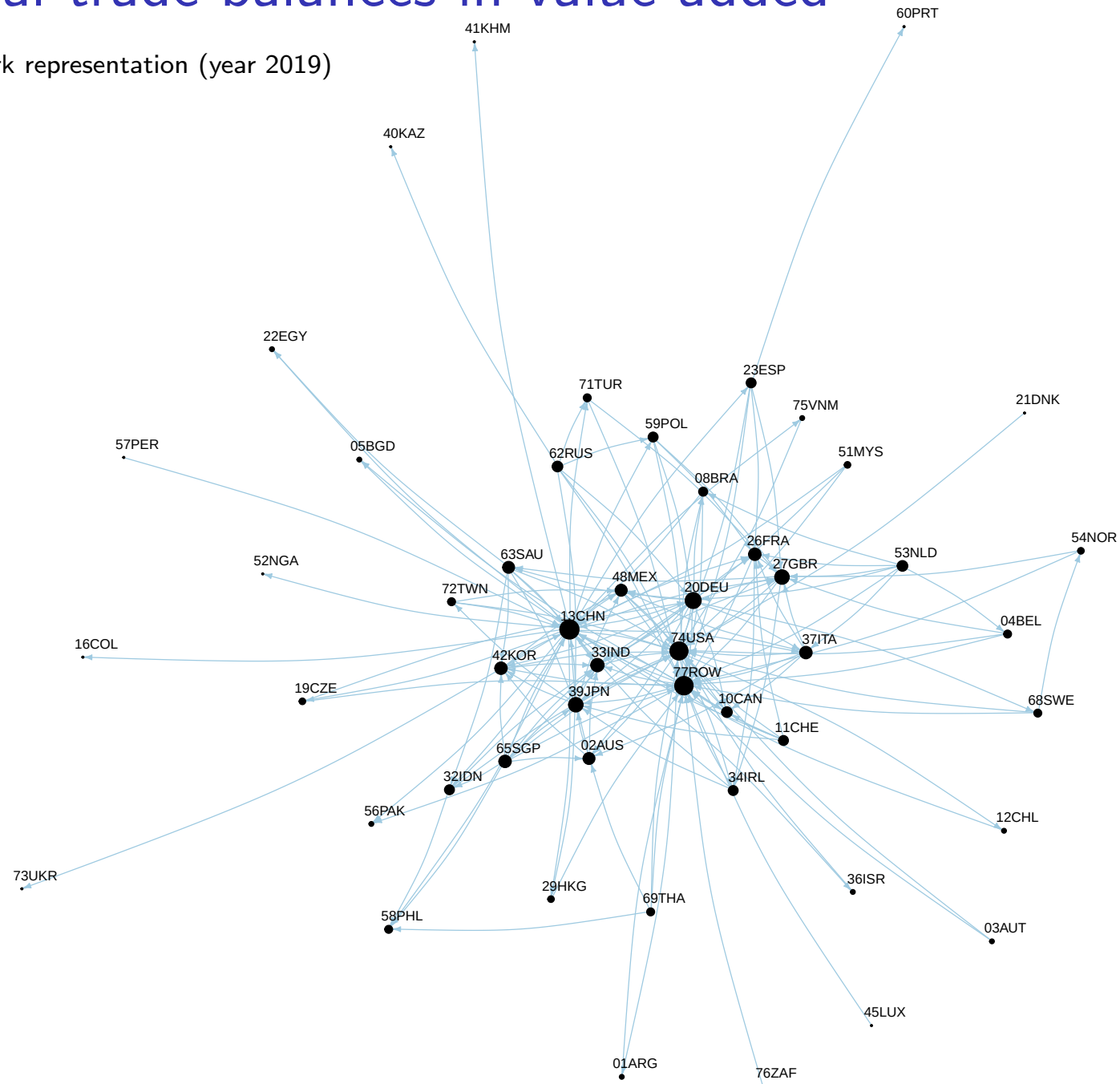
TBVAF (VA flows) - TB (gross flows):

> 0: surplus is higher in VA terms; deficit is smaller in VA terms

< 0: surplus is smaller in VA terms; deficit is higher in VA terms.

Bilateral trade balances in value added

Network representation (year 2019)



Measuring value extraction in GVCs

- Analytical definition of a GVC in I-O terms (Timmer et al., 2013);
- How much value added each country *appropriates/extracts* from the global production process of *one element* of final output i in country C?
- Traditional measure of competitiveness: country share in world gross exports of a product;
- *Novel* measure of 'competitiveness': country share of world income of a GVC (producing *one* final product).

Measuring value extraction in GVCs: final product vector

Sum across output destinations for each element of final demand:

$$\underset{(n\cdot k\times n\cdot k)}{\begin{bmatrix} \boldsymbol{f}_{cc} & \boldsymbol{f}_{cp} & \boldsymbol{f}_{cr} \\ \boldsymbol{f}_{pc} & \boldsymbol{f}_{pp} & \boldsymbol{f}_{pr} \\ \boldsymbol{f}_{rc} & \boldsymbol{f}_{rp} & \boldsymbol{f}_{rr} \end{bmatrix}} \rightarrow \begin{bmatrix} \boldsymbol{f}_{cc}+\boldsymbol{f}_{cp}+\boldsymbol{f}_{cr} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{f}_{pc}+\boldsymbol{f}_{pp}+\boldsymbol{f}_{pr} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{f}_{rc}+\boldsymbol{f}_{rp}+\boldsymbol{f}_{rr} \end{bmatrix} \rightarrow$$

$$\rightarrow \begin{bmatrix} \boldsymbol{f}_{cc}+\boldsymbol{f}_{cp}+\boldsymbol{f}_{cr} \\ \boldsymbol{f}_{pc}+\boldsymbol{f}_{pp}+\boldsymbol{f}_{pr} \\ \boldsymbol{f}_{rc}+\boldsymbol{f}_{rp}+\boldsymbol{f}_{rr} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{c} \\ \boldsymbol{f}_{p} \\ \boldsymbol{f}_{r} \end{bmatrix} = \underset{(n\cdot k\times 1)}{\boldsymbol{f}}$$

to obtain a global final output vector $\boldsymbol{f}$, whose typical element f_i^c contains the total value of final products produced by industry i in country c.

Measuring value extraction in GVCs: GVC income matrix

GVCY matrix: value added content of final *output*

$$\texttt{GVCY} = \widehat{\boldsymbol{v}}\boldsymbol{B}\widehat{\boldsymbol{f}} = \underbrace{\begin{bmatrix} \widehat{\boldsymbol{v}}_c & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widehat{\boldsymbol{v}}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \widehat{\boldsymbol{v}}_r \end{bmatrix}}_{(=\widehat{\boldsymbol{v}})} \underbrace{\begin{bmatrix} \boldsymbol{B}_{cc} & \boldsymbol{B}_{cp} & \boldsymbol{B}_{cr} \\ \boldsymbol{B}_{pc} & \boldsymbol{B}_{pp} & \boldsymbol{B}_{pr} \\ \boldsymbol{B}_{rc} & \boldsymbol{B}_{rp} & \boldsymbol{B}_{rr} \end{bmatrix}}_{(=\boldsymbol{B})} \underbrace{\begin{bmatrix} \widehat{\boldsymbol{f}}_c & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widehat{\boldsymbol{f}}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \widehat{\boldsymbol{f}}_r \end{bmatrix}}_{(=\widehat{\boldsymbol{f}})}$$

$$\underset{(n\cdot k)\times(n\cdot k)}{\texttt{GVCY} = \widehat{\boldsymbol{v}}\boldsymbol{B}\widehat{\boldsymbol{f}}} = \begin{bmatrix} \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cc}\widehat{\boldsymbol{f}}_c & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cp}\widehat{\boldsymbol{f}}_p & \widehat{\boldsymbol{v}}_c\boldsymbol{B}_{cr}\widehat{\boldsymbol{f}}_r \\ \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\widehat{\boldsymbol{f}}_c & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pp}\widehat{\boldsymbol{f}}_p & \widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pr}\widehat{\boldsymbol{f}}_r \\ \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rc}\widehat{\boldsymbol{f}}_c & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rp}\widehat{\boldsymbol{f}}_p & \widehat{\boldsymbol{v}}_r\boldsymbol{B}_{rr}\widehat{\boldsymbol{f}}_r \end{bmatrix}$$

Country P's GVC income of value chain i in country C:

$$\texttt{GVCY}_{ic}(p) = \mathbf{1}^T\widehat{\boldsymbol{v}}_p\boldsymbol{B}_{pc}\mathbf{1}_i f_i^c = v_1^p b_{1i}^{pc} f_i^c + v_2^p b_{2i}^{pc} f_i^c$$

Shares of world GVC income

World GVC income shares of macro areas by product (in %)

(Year 1995)

		US	EU	RoGN	CHN	RoGS
A01AGR	Agriculture & Forestry	8.7	21.1	15.2	10.1	44.9
B03FIS	Fishing	2.3	11.7	21.9	13.0	51.2
B05MEN	Mining - Energy	4.3	16.4	14.3	5.2	59.8
B07MNE	Mining - Non-Energy	8.1	14.9	13.6	8.3	55.1
B09MSS	Mining - Support	41.4	7.6	10.4	0.2	40.4
C10FOD	Food products	17.7	27.9	24.6	3.9	25.8
C13TEX	Textile products	16.3	28.1	22.9	7.0	25.7
C16WOD	Wood products	29.8	37.1	14.2	3.4	15.5
C17PAP	Paper & Printing	25.0	27.4	30.5	1.6	15.4
C19PET	Petroleum products	16.8	15.7	20.0	1.5	46.0
C20CHM	Chemical products	25.7	29.0	22.0	2.8	20.6
C21PHA	Pharmaceuticals	22.7	31.7	28.1	2.3	15.3
C22RBP	Rubber & Plastics	19.5	28.7	33.1	2.9	15.8
C23NMM	Non-metal Min. Prod.	11.9	35.3	20.5	10.6	21.6
C24MET	Basic metals	17.6	18.7	30.2	8.2	25.3
C25FBM	Fabricated metal prod.	18.2	40.1	22.6	3.0	16.1
C26CEQ	ICT Equip.	26.7	15.1	41.5	2.8	13.9
C27ELQ	Electrical Equip.	14.7	24.8	41.9	3.6	15.0
C28MEQ	Mechanical Equip.	18.8	29.1	37.4	4.6	10.2
C29MTR	Motor vehicles	25.1	30.3	29.6	1.5	13.5
C30TRQ	Transport Equip.	30.3	21.2	31.8	1.8	14.9
C31OTM	Other Manufacturing	23.7	37.4	22.8	2.0	14.1
D35EGC	Electricity & Gas	20.9	27.6	28.9	4.0	18.6
E36WRE	Water & Recycling	9.6	27.9	40.9	3.5	18.0
F41CON	Construction	17.1	25.6	36.4	4.0	16.9
G45WRT	Trade	24.7	27.9	28.7	1.9	16.8
H49LTR	Land transport	13.9	26.5	32.5	1.5	25.7
H50WTR	Water transport	14.1	26.4	34.7	2.5	22.3
H51ATR	Air transport	31.6	21.8	22.9	1.6	22.1
H52STR	Logistics	7.3	30.8	46.3	1.2	14.4
H53POS	Postal & Courier	25.4	23.1	34.3	2.4	14.9
I55HTR	Hotels & Restaurants	23.1	27.5	32.1	1.5	15.9
J58PAB	Media services	46.3	24.7	21.2	0.6	7.2
J61TLC	Telecommunications	30.7	22.5	25.0	0.7	21.1
J62ITS	IT Services	25.9	21.3	37.0	1.2	14.5
K64FIN	Finance	34.3	20.2	31.3	0.9	13.4
L68REA	Real Estate	28.2	24.5	32.9	0.9	13.5
M69KIS	Knowledge services	28.0	28.8	32.9	2.6	7.8
N77ASS	Admin services	18.8	30.3	36.8	2.7	11.4
O84PUB	Public Admin	36.0	23.9	22.6	1.3	16.3
P85EDU	Education	30.8	26.2	24.8	1.5	16.7
Q86HTH	Health & Social Work	32.6	28.7	27.5	1.0	10.1
R90ART	Arts & Entertainment	21.5	28.2	36.4	0.9	13.0
S94OTS	Other services	23.1	26.6	23.3	1.9	25.2

(Year 2019)

	US	EU	RoGN	CHN	RoGS
A01AGR	6.9	9.8	6.3	22.0	55.1
B03FIS	1.7	4.1	6.4	28.4	59.4
B05MEN	24.0	3.7	11.1	2.6	58.6
B07MNE	24.7	8.8	11.9	5.4	49.2
B09MSS	61.7	2.5	6.3	1.3	28.1
C10FOD	15.0	16.2	12.5	18.2	38.1
C13TEX	4.4	12.7	6.5	35.1	41.3
C16WOD	21.1	30.1	9.6	8.9	30.3
C17PAP	19.2	20.7	11.3	20.6	28.2
C19PET	21.4	9.0	13.3	5.1	51.2
C20CHM	25.0	20.8	13.2	9.4	31.7
C21PHA	28.8	24.5	15.5	15.6	15.6
C22RBP	22.1	22.9	15.1	11.2	28.7
C23NMM	14.1	24.6	10.2	12.1	39.0
C24MET	13.4	11.3	14.2	8.4	52.8
C25FBM	14.9	22.5	10.9	28.4	23.3
C26CEQ	18.9	11.1	18.8	29.8	21.5
C27ELQ	9.8	17.5	16.9	32.4	23.5
C28MEQ	13.1	20.2	19.1	31.5	16.1
C29MTR	17.3	23.0	17.4	20.8	21.5
C30TRQ	26.4	18.6	16.3	19.3	19.5
C31OTM	18.6	24.5	13.5	19.7	23.7
D35EGC	16.0	16.7	15.0	23.8	28.5
E36WRE	12.4	22.3	27.6	15.1	22.6
F41CON	14.3	13.1	15.6	32.3	24.7
G45WRT	28.4	19.2	16.1	9.8	26.6
H49LTR	17.0	16.5	14.6	14.4	37.5
H50WTR	14.6	19.6	17.6	13.9	34.4
H51ATR	28.1	15.1	14.1	6.0	36.8
H52STR	10.3	30.0	22.6	10.7	26.6
H53POS	23.1	17.6	14.0	12.2	33.1
I55HTR	26.9	20.2	17.0	9.9	26.0
J58PAB	46.6	17.5	16.0	6.8	13.1
J61TLC	27.6	13.1	17.6	12.5	29.2
J62ITS	29.5	19.3	17.4	15.6	18.3
K64FIN	36.1	13.5	17.7	12.1	20.7
L68REA	30.8	18.8	21.6	9.8	19.0
M69KIS	29.7	24.2	18.9	14.4	12.7
N77ASS	20.4	25.5	14.7	23.1	16.2
O84PUB	33.0	15.3	15.0	12.5	24.1
P85EDU	30.2	16.2	15.8	13.9	24.0
Q86HTH	36.3	21.0	20.5	8.4	13.8
R90ART	27.0	25.1	19.6	9.2	19.1
S94OTS	26.5	17.8	13.8	12.0	30.0

(2019 - 1995)

	US	EU	RoGN	CHN	RoGS
A01AGR	-1.8	-11.3	-8.9	11.9	10.2
B03FIS	-0.6	-7.6	-15.5	15.4	8.2
B05MEN	19.7	-12.6	-3.2	-2.6	-1.2
B07MNE	16.5	-6.1	-1.7	-2.8	-5.9
B09MSS	20.3	-5.1	-4.1	1.1	-12.2
C10FOD	-2.8	-11.7	-12.1	14.3	12.2
C13TEX	-11.9	-15.4	-16.4	28.1	15.6
C16WOD	-8.7	-7.0	-4.6	5.5	14.8
C17PAP	-5.8	-6.7	-19.2	19.0	12.8
C19PET	4.6	-6.7	-6.7	3.6	5.2
C20CHM	-0.7	-8.2	-8.8	6.6	11.2
C21PHA	6.2	-7.2	-12.6	13.3	0.3
C22RBP	2.5	-5.8	-18.0	8.3	12.9
C23NMM	2.1	-10.8	-10.3	1.5	17.4
C24MET	-4.3	-7.4	-16.0	0.2	27.4
C25FBM	-3.3	-17.6	-11.7	25.4	7.2
C26CEQ	-7.8	-4.0	-22.7	26.9	7.6
C27ELQ	-4.9	-7.3	-25.0	28.8	8.4
C28MEQ	-5.6	-8.8	-18.4	26.9	6.0
C29MTR	-7.8	-7.3	-12.2	19.3	8.0
C30TRQ	-3.9	-2.6	-15.5	17.4	4.6
C31OTM	-5.1	-12.9	-9.3	17.8	9.6
D35EGC	-4.9	-10.9	-14.0	19.7	10.0
E36WRE	2.8	-5.6	-13.3	11.5	4.6
F41CON	-2.8	-12.5	-20.8	28.3	7.8
G45WRT	3.6	-8.7	-12.6	7.9	9.7
H49LTR	3.1	-10.0	-17.8	12.9	11.8
H50WTR	0.4	-6.8	-17.2	11.4	12.1
H51ATR	-3.5	-6.7	-8.8	4.3	14.6
H52STR	2.9	-0.8	-23.8	9.5	12.2
H53POS	-2.2	-5.5	-20.3	9.8	18.2
I55HTR	3.8	-7.2	-15.2	8.4	10.1
J58PAB	0.4	-7.2	-5.3	6.3	5.8
J61TLC	-3.1	-9.4	-7.4	11.8	8.1
J62ITS	3.6	-2.1	-19.7	14.4	3.7
K64FIN	1.8	-6.7	-13.7	11.2	7.4
L68REA	2.7	-5.7	-11.4	8.9	5.5
M69KIS	1.8	-4.6	-13.9	11.8	4.9
N77ASS	1.6	-4.8	-22.1	20.5	4.8
O84PUB	-3.0	-8.6	-7.5	11.2	7.8
P85EDU	-0.7	-10.0	-9.0	12.4	7.3
Q86HTH	3.6	-7.7	-7.0	7.4	3.7
R90ART	5.5	-3.1	-16.9	8.3	6.1
S94OTS	3.3	-8.8	-9.5	10.1	4.9

Source: Author's computation based on OECD-ICIO (2023 Edition)

The practitioner's corner

Principal indicators ready to use: OECD Trade in Value Added (TiVA) 2023 Ed. (here)

Global IRIO databases available:

- World Input-Output Database (here)
- OECD Inter-Country Input-Output (ICIO) Database (here)
- EU-FIGARO (here)
- GLORIA (here)
- EXIOBASE (here)

The practitioner's corner (cont'd)

You may also use the data toolkit accompanying this lecture (`tb_data_lecture.xlsx`):

	A	B
1	*Table of Contents: Measuring globalisation and GVCs through Input-Output techniques*	
2	(Ariel L. Wirkierman)	
3		
4	Worksheet	Content
5	tb_iso_regions	Allocation of countries into macro areas
6	tb_ind	Industry classification
7	tb_IRIO_reg	Global Input-Output scheme for 5 macro areas (all values in 10^9 current USD)
8	tb_IRIO_diag_reg	Domestic output components of Global Input-Output scheme for 5 macro areas (all values in 10^9 current USD)
9	tb_TB_cp	Bilateral trade balance matrices by year; gross flows; row view = X - M; column view = M - X (all values in 10^6 current USD)
10	tb_TBVAF_cp	Bilateral trade balance matrices by year; value added flows; row view = X - M; column view = M - X (all values in 10^6 current USD)
11	tb_GVCY_c_by_i	Matrix of GVC income of product in column captured by country in row by year (all values in 10^6 current USD)
12	tb_VAX_VAF	*Indicators computed with the VAX and VAF matrices*
13	DVAX1_c	Domestic value added content of own gross exports (in 10^6 current USD)
14	DVAX2_c	Domestic value added content of foreign gross exports (in 10^6 current USD)
15	FVAX_c	Foreign value added content of gross exports (in 10^6 current USD)
16	EXP_c	Gross exports (in 10^6 current USD)
17	SH_DVAX1_c	= DVAX1_c / EXP_c
18	SH_DVAX2_c	= DVAX2_c / EXP_c
19	SH_FVAX_c	= FVAX_c / EXP_c
20	GVC_part_c	GVC participation index (SH_DVAX2_c + SH_FVAX_c)
21	DVAF_c	Domestic value added induced by foreign final demand (in 10^6 current USD)
22	FVAF_c	Foreign value added induced by domestic final demand (in 10^6 current USD)
23	TBVAF_c	= DVAF_c - FVAF_c
24	Y_c	Gross Value added and net taxes on products (in 10^6 current USD)
25	FD_c	Domestic final demand (in 10^6 current USD)
26	SH_DVAF_c	= DVAF_c / Y_c
27	SH_FVAF_c	= FVAF_c / FD_c
28	F_c	Net product (in 10^6 current USD)

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