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(Dis)Solving the Zero Lower Bound Equilibrium through Income Policy

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Abstract

We investigate the possibility to reflate an economy experiencing a long-lasting zero lower bound episode with subdued or negative inflation by imposing a minimum level of wage inflation. Our proposed income policy relies on the same mechanism behind past disinflationary policies, but it works in the opposite direction. It is formalized as a downward nominal wage *growth* rigidity (DNWGR), such that wage inflation cannot be lower than a fraction of the inflation target. This policy allows dissolving the zero lower bound steady state equilibrium in an OLG model featuring "secular stagnation" and in an infinite-life model, where this equilibrium emerges due to deflationary expectations.

Keywords: Zero lower bound, Wage indexation, Inflation expectations.
JEL classification: E31, E52, E64.

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1 Introduction

In most advanced economies, the zero lower bound (ZLB) on nominal rates has been binding since the onset of the Great Recession, and inflation has been steadily below the target level. Against this backdrop, in the pre-COVID 19 era, some economists made a case for income policies, as part of a broad policy package including monetary and fiscal stimuli, to reflate the economy. We lay the theoretical foundation for this policy proposal by defining an income policy that imposes a minimum level of wage inflation and by investigating its effectiveness in tackling the ZLB problem.

We explain the essence of our "*reflationary*" income policy through a past *dis-inflationary* income policy, which relied on the *same* mechanism, but worked in the *opposite* direction. In Italy, the inflation rate was about 6% in the early 1990s and needed to decrease by approximately 4% in a few years to satisfy the Maastricht inflation criterion. Italy met the challenge through a coordinated effort by different policymakers and social actors. Still, the Protocol signed by the employers and trade-union organizations on 23 July 1993 was the cornerstone for the structural reduction of inflation. It marked the definite dismantling of the automatic indexation to the past inflation mechanism (*scala mobile*), and established the price inflation expected and targeted by the government (*tasso d'inflazione programmata*) as a common reference for the indexation of national collective contracts.[1] The

[1] The "Protocol on Incomes and Employment Policy, on Contractual Arrangements, on Labor Policies and Support for the Production System" (*Protocollo sulla politica dei redditi e dell'occupazione, sugli assetti contrattuali, sulle politiche del lavoro e sul sostegno al sistema produttivo*) was drafted on 3 July 1993. While the so-called *scala mobile* had ceased to operate already in 1992, wage-setting was still very much backward-looking. The Protocol represented the last stage of a long-term strategy against inflation designed by Italian institutions. It was launched in the mid-1980s and implemented through exchange rate policies, monetary restraint, and income policies (Gressani et al.,

main channel that led to the successful disinflation was the realignment of inflation expectations to the target level chosen by the government (Fabiani et al., 1998; Destefanis et al., 2005).

This historical example shows that de-indexing the economy is an effective way to tackle inflation. The Italy's problem was a problem of "de-indexing" the economy by de-indexing the wage bargaining process and thus breaking the wage-price inflation spiral. This type of income policy was popular at the time, and many cases show that they could be a very efficient way to disinflate the economy.[2]

The other side of the coin could be that "re-indexing" the economy is an effective way to tackle deflation. The idea is that past disinflationary income policies were thought to stop the *upward* inertia in the behavior of inflation, the so-called wage-price spiral. The problem in a ZLB, or in the path that lead to the ZLB, derives from the same logic, but it is a spiral *downward* rather than upward. Although demand-side measures can be effective in output stabilization, as proven during the Great Recession, they cannot address and solve the ZLB problem without the support of supply-side interventions such as income policies. Indeed, the ZLB, and the often corresponding deflation or too low inflation problem, is a "nominal" problem. Once we see the problem in this way, we can think about it as a "reflation" problem, which is just the opposite of a disinflation.

This paper shows that the same income policies of the '80s and '90s can be effectively used the other way round, that is in the opposite direction, to engineer a reflation and avoid the ZLB. We propose to impose a lower bound on wage inflation:

1988).

[2]For instance, the package of disinflationary policies implemented by Israel and France in the mid-1980s included income policies that played the pivotal role of defining a new nominal anchor (da Silva and Mojon, 2019).

an income policy based on a downward nominal wage growth rigidity (DNWGR) such that wage inflation cannot be lower than a fraction of the intended inflation target. With the DNWGR constraint, there will always exist a level of inflation target that eradicates the ZLB equilibrium.

We show how this income policy works in two very different frameworks: the model of Eggertsson et al. (2019) (EMR, henceforth) and that of Schmitt-Grohé and Uribe (2017) (SGU henceforth). EMR develop an overlapping generation (OLG) model of secular stagnation, in which a ZLB equilibrium arises when the natural interest rate is negative. SGU build an infinite-life representative agent model, in which a ZLB equilibrium can arise due to expectations of deflation, as in Benhabib et al. (2001). By investigating the effectiveness of our reflationary income policy in these two frameworks, we test its "robustness", namely its capacity to address the ZLB problem independently of its source (Bilbiie, 2018).

Both papers also feature a downward nominal wage rigidity (DNWR) constraint to be replaced by our DNWGR, which is a form of wage indexation to the inflation target.[3] Although our policy would also work if the economy was trapped in a ZLB/deflationary equilibrium without a binding DNWR to start with and thus without unemployment, we view the DNWR as pervasive in modern economies, imposing cost in terms of output/employment under low inflation. Instead, wage indexation, independently of its focal point, is less pervasive.[4] As a consequence,

[3]Regardless of the focal point in wage negotiations, wage indexation and downward real wage rigidity (DRWR) are often used as synonyms for asymmetric indexation clauses that can be revised only upward. Instead, we prefer to define our income policy as a DNWGR, to distinguish it from wage indexation to past inflation, which will be referred to as DRWR.

[4]Holden and Wulfsberg (2009) find a greater extent of DNWR than of wage indexation in the OECD countries. A significant degree of DNWR persisted even in the face of the Great Recession (Branten et al., 2018; Fallick et al., 2020), leading to costs in terms of employment, as documented, among the others, by Kurmann and McEntarfer (2019).

the implementation of our income policy requires a shift in the prevailing wage rigidity from DNWR to wage indexation, to be supported by proper institutional arrangements as for past disinflationary policies in Italy.

Our reflationary income policy eliminates the ZLB equilibrium in both the theoretical frameworks considered, provided that the inflation target is sufficiently high. If wage inflation is high enough, then agents cannot coordinate on a deflationary or a secular stagnation equilibrium, because expectations of deflation, or low inflation, and ZLB are not consistent with rational expectations. Our mechanism has the same flavour as the Italian case, but upside-down. In equilibrium, the DNWGR constraint does not bind; hence, it is not the case that it is mechanically imposed. Moreover, both price and wage inflation are equal to the intended target, and there is full employment in the unique equilibrium that survives. Our reflationary income policy acts as a coordination device that destroys the bad ZLB equilibrium.

Our paper relates to the policy debate, in the pre-COVID 19 era, regarding the possible implementation of income policy to reflate an economy in ZLB. Arbatli et al. (2016) advocate income policy as a "fourth arrow" to the "Abenomics". However, they simulate the IMF Flexible System of Global Models (FSGM) without formalizing a proper income policy but adding shocks to expectations of both price and wage inflation. Instead, da Silva and Mojon (2019) propose an increase in the nominal unit labor costs consistent with the prevailing inflation target of 2% in advanced economies. Still, they do not investigate the implications of this proposal, in particular its effectiveness, explicitly.

On the other hand, this paper is linked to the enormous literature on ZLB.[5]

[5] This literature focuses mostly on dynamics, studying the cost of the ZLB constraint on monetary policy (e.g., Gust et al., 2017), and how to coordinate expectations to escape from the liquidity trap

Specifically, Mertens and Ravn (2014) and Bilbiie (2018) study the effects of different policies around different equilibria, depending on whether the liquidity trap is fundamental or expectation-driven. They find that the policies beneficial in the former are detrimental in the latter and vice versa. In contrast, our reflationary income policy is "robust" because it can address and solve the ZLB problem independently of its source. In this respect, our work is also related to that of Cuba-Borda and Singh (2019), who find a similar result for minimum wage policy, considering a unified framework that simultaneously accommodates the secular stagnation hypothesis and the expectation-driven liquidity trap. Although minimum wage and reflationary income policies are both robust, our policy is more effective because it *eliminates* both the expectations trap and the secular stagnation equilibrium, imposing a floor on the *growth rate* of nominal wage. In contrast, the minimum wage policies can only *mitigate* the secular stagnation one, imposing a floor on the nominal wage *level*. Moreover, in Cuba-Borda and Singh (2019), as in EMR and SGU, increasing the inflation target cannot eliminate any of the two bad equilibria. Instead, it does so in our framework, and thus there is no issue of credibility of the target due to the coexistence of multiple steady states.

The paper proceeds as follows. Section 2 presents how our policy would work in the EMR model, while Section 3 does the same in the SGU model. Section 4 concludes the paper. Appendices A.1 and A.2 spell out the derivations, respectively, of the EMR and SGU models, in particular, those regarding the aggregate demand that is not central in our analysis.

(e.g., Benhabib et al., 2002).

2 Reflation in the EMR OLG model

We study a three-period OLG economy, in which the equilibrium real interest rate, r_t, is endogenously determined in the market for assets, and it coincides with the "natural" interest rate r_t^f when output is at the potential level Y^f. Figure 1 conveniently depicts the steady state relationships implied by this model, using an aggregate demand (*AD*) and aggregate supply (*AS*) diagram. Both AS and AD curves are characterized by two regimes, governed respectively by the DNWR and the ZLB, and so they exhibit a kink.

The production technology of firms exhibits decreasing returns to labor, $Y_t = L_t^{\alpha}$ with $0 < \alpha < 1$. The labor market operates under perfect competition. However, workers are unwilling to supply labor for a nominal wage lower than a minimum level:

$$W_t = \max\left\{W_t^*, W_t^{flex}\right\}. \tag{1}$$

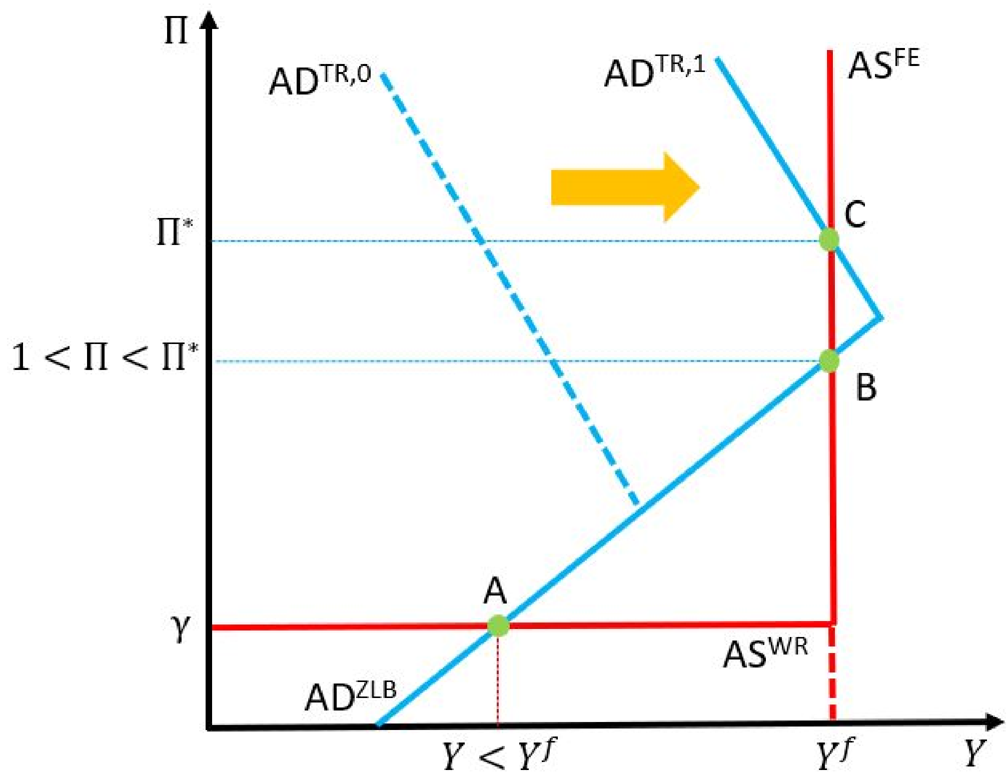
W_t^{flex} is the "flexible" wage compatible with full employment $\bar{L}$, and W_t^* is the lower bound on the nominal wage *level* that is proportional to that in the previous period (Schmitt-Grohé and Uribe, 2016):[6]

$$W_t^* = \gamma W_{t-1}, \tag{2}$$

where $0 < \gamma \leq 1$. The labor market does not necessarily clear because of downwardly rigid wages. If labor market clearing requires a wage W_t larger than γW_{t-1}, the nominal wage is flexible and the labor market clears at full employment, $L_t = \bar{L}$.

[6] EMR originally assume $W_t^* = \gamma W_{t-1} + (1-\gamma) W_t^{flex}$. Our modification does not alter the model, but allows for better comparability with our income policy and the DNWR in SGU.

Figure 1: Steady State equilibria in the EMR model



The AS curve is accordingly vertical at the potential output $Y^f = \bar{L}^{\alpha}$. In contrast, if labor supply exceeds labor demand at the wage $W_t = \gamma W_{t-1} > W_t^{flex}$, the wage cannot decrease further because of the DNWR constraint, so that involuntary unemployment arises, $L_t < \bar{L}$. In this case, the AS is thus flat at the wage/price inflation $\Pi^W = \Pi = \gamma$ for $\forall L \leq \bar{L}$, with the level of employment/output that is demand-determined along the AS^{WR}.

The central bank follows a standard Taylor rule that responds only to inflation, and it is subject to the ZLB constraint:

$$1+i_t = \max\left[1, \left(1+r_t^f\right)\Pi^*\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right], \tag{3}$$

where $\phi_\pi > 1$. $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate at time t and Π^* is the gross inflation target. Whether or not the ZLB constraint (3) is binding defines the two regimes for the AD curve. When the ZLB does not bind, the AD defines a negative relationship between inflation and output, represented by the downward-sloping curve AD^{TR} in Figure 1. By contrast, when central bank hits the ZLB and thus $i = 0$, there exists a positive relationship between steady state inflation and output, and we depict it as an upward-sloping AD curve, AD^{ZLB}, in Figure 1.[7] We denote Π^{kink} the inflation rate at which AD^{TR} and AD^{ZLB} cross, which is

$$\Pi^{kink} = \left[\frac{1}{(1+r^f)}\right]^{\frac{1}{\phi_\pi}} \Pi^{*\frac{\phi_\pi-1}{\phi_\pi}}. \tag{4}$$

Π^{kink} determines when the ZLB becomes binding.

To prepare the ground for the intuition of our main result, Figure 1 plots how the AD curve moves with the inflation target Π^*, as depicted by the shift from $AD^{TR,0}$ to $AD^{TR,1}$. The higher the inflation target, the lower the risk of hitting the ZLB for a given natural interest rate r^f in (3). Therefore, a higher Π^* shifts out the kink in the AD, and thus the downward-sloping AD curve, but it does not affect the upward-sloping one.

[7]Figure 1 follows Figure 6 Panel A in EMR and the discussion therein in Section VI, p. 25. As EMR, we depict AD^{TR} as linear in Figure 1 for clarity, despite it being non-linear (see Appendix A.1). We will do the same for AD^{TR} in the SGU model. None of the results obviously depends on this.

The crossing between the AS and the AD curves identifies a steady state in Figure 1. A "secular stagnation" equilibrium always arises for $r^f < 0$, but there can be two different cases depending on the inflation target. First, in the case corresponding to the dashed line $AD^{TR,0}$, there is a unique steady state at point A, which is a demand-determined and stagnant steady state (secular stagnation) because ZLB and DNWR are both binding. Second, in the case corresponding to the solid line $AD^{TR,1}$, there are three different steady states: (A) the *ZLB-U* equilibrium just described featuring binding ZLB, inflation below the target and unemployment: $i = 0, \Pi = \gamma < \Pi^*, Y < Y^f$; (B) a *ZLB-FE* equilibrium featuring binding ZLB, inflation below the target and full employment: $i = 0, 1 < \Pi = \frac{1}{1+r^f} < \Pi^*, Y = Y^f$; (C) a *TR-FE* equilibrium featuring a positive nominal interest rate, inflation below the target and full employment: $i > 0, \Pi = \Pi^*, Y = Y^f$.[8] [9]

As explained earlier, an increase in the inflation target moves AD^{TR}, but moves neither the AD^{ZLB} nor the AS. Hence, if the natural real interest rate is negative, a *ZLB-U* equilibrium always exists no matter what the inflation target is. EMR look at alternative options, like fiscal policy, to raise r^f to positive values because in their model monetary policy is powerless, but this is no longer the case with our reflationary income policy.[10]

[8] EMR show that the equilibria *ZLB-U* and *TR-FE* are determinate, while the equilibrium *ZLB-FE* is an indeterminate, but not deflationary, steady state á la Benhabib et al. (2001). They show it for a different DNWR specification (see footnote 6), but these results still hold in the simpler specification of this Section. Results are available upon request.

[9] For $\gamma = 1$, the DNWR coincides with the minimum wage policy of Cuba-Borda and Singh (2019), and it cannot eliminate, as well as the secular stagnation equilibrium, the *ZLB-FE* one. Hence, that policy seems ineffective if the equilibrium á la Benhabib et al. (2001) features positive, below the target inflation as in the EMR model.

[10] While there is always a minimum level of public debt that makes $r^f > 0$ and eliminates the *ZLB-U* equilibrium, this value might be very high and not necessarily sustainable/achievable, as shown by EMR in their quantitative exercise and witnessed by the recent experience of Japan. We study this case in Appendix A.1.4, where we show that our income policy is complementary not only to the

2.1 Dissolving the ZLB Equilibrium

We now present an income policy capable of avoiding a secular stagnation even if $r^f < 0$. As explained in the Introduction, the secular stagnation equilibrium *ZLB-U* vanishes with our policy, which is a simple modification of W_t^* in equation (1) to

$$W_t^* = \delta \Pi^* W_{t-1}, \tag{5}$$

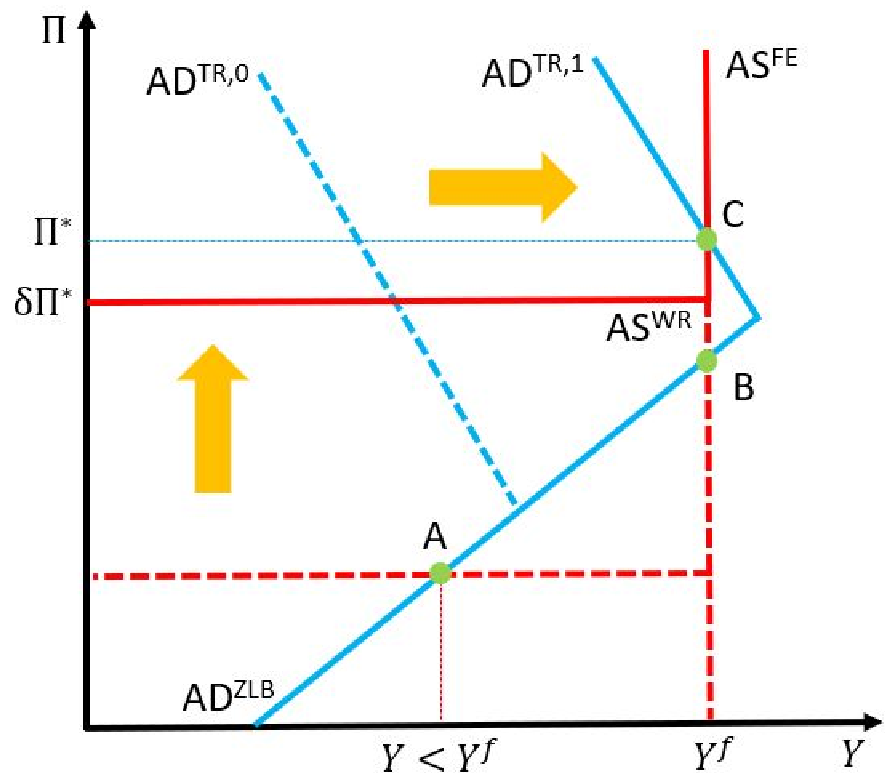
where $0 < \delta \leq 1$. From an economic point of view, (5) implies that wage inflation cannot be lower than a certain fraction δ of the inflation target Π^*. Wage indexation to target inflation captures the idea behind the disinflationary policies in Italy. However, while there the goal was to put a *ceiling* on the pressure for wage increases to *decrease* the rate of inflation, here the goal is to put a *floor* on wage deflation to *increase* the rate of inflation.

Although the modification in (1) seems minimal, it underlies a substantial change. Equation (2) establishes a lower bound on the nominal wage *level*, with γ measuring how much, if anything, the wage can be cut downward. In contrast, (5) imposes a lower bound to the nominal wage *growth rate*, with δ being the degree of wage indexation to the inflation target. Therefore, the implementation of our reflationary income policy requires a switch from DNWR to DNWGR as prevailing wage rigidity. The DNWR here simply disappears and is replaced by the DNWGR. We consider the coexistence of the two in Appendix A.1.5, showing that our results/propositions are unaffected.[11]

monetary policy but also to the fiscal one.

[11] In principle, average and targeted inflation could affect the degree of wage indexation and DNWR. However, the empirical evidence is contrasting and variable across countries and time (see, e.g., Messina and de Galdeano, 2014). On the other hand, the effect of *exogenous* variables expressing

Figure 2: Unique steady state equilibrium in the EMR model with DNWGR



From an analytical point of view, comparing Figure 2 with Figure 1 reveals how this modification changes the results in the previous section. The main point is that (5) *makes the AS curve to shift with the inflation target*, because the AS^{WR} curve is now equal to $\delta\Pi^*$, rather than γ, as in the EMR case. Hence, an increase in the inflation target shifts the AS^{WR} curve upward. As the AD curve is unchanged with respect to the previous section, raising the inflation target shifts out AD^{TR}, as

the state of industrial relations (e.g., union density and coverage) on the degree of DNWR and wage indexation is firmly established on empirical grounds (Dickens et al., 2007; Holden and Wulfsberg, 2008, 2009). We view the change in the prevailing wage rigidity accordingly as an exogenous process summarized by changes in the corresponding parameters, and we leave the study of how realized/target inflation influences wage indexation for future research.

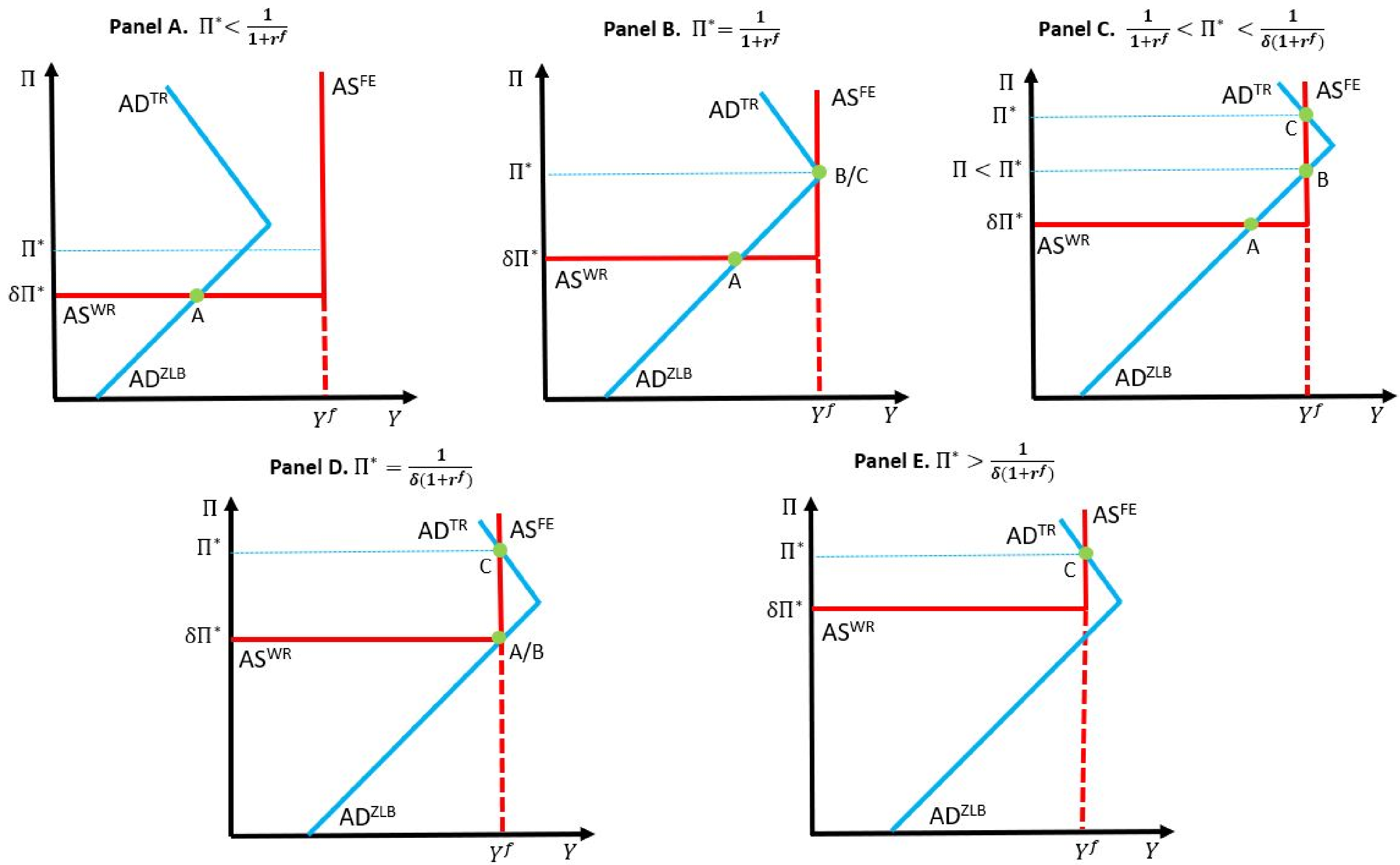
in Figure 1. We are now in the position to state our main result in the following proposition.

Proposition 1. *Assume* $r^f < 0$ *and* $\delta \leq 1$. *Then, if* $\Pi^* > \frac{1}{\delta(1+r^f)}$, *there exists a unique, locally determinate,* $TR-FE$ *equilibrium, where the ZLB is not binding, the inflation rate is equal to the target and output is at full employment, i.e.,* $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.

In other words, there always exists a sufficiently high level of the inflation target, Π^*, such that the unique and locally determinate equilibrium features full employment and inflation at the target without binding ZLB. While the formal proof of Proposition 1 is in Appendix A.1.3, Figure 3 displays the intuition very clearly.[12] It shows five different panels, each for different ranges of values of the inflation target. As the inflation target increases, the economy moves from Panel A to Panel E. The key thing to note is that the *AD* curve moves as described in the previous section, and now also the AS^{WR} shifts upward. For a sufficiently high inflation target, the economy reaches the situation in Panel E, where only the $TR-FE$ equilibrium exists. Therefore, the secular stagnation equilibrium, $ZLB-U$, disappears if $\Pi^* \geq \left[\delta(1+r^f)\right]^{-1}$.

[12]Figure 3 refers to the general case $\delta < 1$. We report the limiting case $\delta = 1$ in Appendix A.1.3, where we also discuss in more detail the five panels in the figure and the corresponding equilibria.

Figure 3: Steady state equilibria in the EMR model with DNWGR



Contrary to EMR where monetary policy is powerless, now monetary policy can wipe out the ZLB equilibrium by choosing an adequate inflation target. However, the degree of wage indexation, δ, also plays a crucial role: the higher this parameter, the lower the inflation target necessary to prevent binding ZLB. Hence, not only wage indexation to targeted inflation has to prevail over DNWR, but it has to be sufficiently widespread *per se* to make the absence of binding ZLB compatible with price stability. We confirm this result in the extended model of Appendix A.1.5, which features both DNWR and DNWGR, as well as in the SGU model below. Moreover, we will discuss below the institutional arrangements within our policy can take place to become the predominant form of wage rigidity and be largely widespread (high δ).

Finally, there is another important implication of our proposed policy with respect to EMR, which we summarize in the next proposition.

Proposition 2. *Assume $r^f < 0$, $\delta \leq 1$, and that the economy is trapped in a secular stagnation equilibrium, $ZLB - U$ (Panel A). Then, an increase in the inflation target is always beneficial, in the sense that steady state output and inflation increase, irrespective of this increase is sufficient or not to escape from the secular stagnation.*

Any, however small, increase in the target shifts the AS^{WR} upward, and thus it moves the secular stagnation equilibrium along the AD^{ZLB} increasing the level of output and inflation. This is depicted in Figure 3, where the $ZLB - U$ equilibrium A in Panel A moves up in Panels B, C and D. This does not happen in the EMR model. In Figure 1 both AD^{ZLB} and AS^{WR} curves do not change with the inflation target. As a result, a mild increase in the target does not affect the secular stagnation equilibrium $ZLB-U$ at point A, capturing Krugman's (2014) idea of "timidity trap". Only sufficiently large changes in the target make the $TR-FE$ equilibrium to appear.[13] Our model has a similar flavour, but has a quite different implication: while it is still true that the policy is subject to a "timidity trap" to escape from the secular stagnation, in the sense that the inflation target should be sufficiently high to avoid it, an increase in the target is always beneficial.

3 Reflation in the SGU infinite-life model

We now turn to a different model and source of binding ZLB, which is a negative shock to inflation expectations. As the logic is very similar in this case, we still convey the model mostly by using figures and put most of the derivations in the appendix.

SGU employ a flexible-price, infinite-life representative agent model to study the dynamics leading to a liquidity trap and a jobless recovery.[14] With respect to the model in the previous section, they also employ a different specification of the DNWR constraint

$$W_t \geq \gamma_0 \left(1-u_t\right)^{\gamma_1} W_{t-1} = \gamma_0 \left(\frac{L_t}{\bar{L}}\right)^{\gamma_1} W_{t-1}. \tag{6}$$

The DNWR implies that the lower bound on the nominal wage depends on the level

[13] *"Small changes in the inflation target have no effect, capturing Krugman's observation of the "law of the excluded middle" or "timidity trap" when trying to explain why the Japanese economy might not respond to a higher inflation target announced by the Bank of Japan unless it was sufficiently aggressive."* (EMR, p.3).

[14] Compared to the original model in SGU, we abstract from growth and fiscal policy, and we assume a logarithmic utility function instead of a more general CCRA specification. Our results are unaffected by this modification.

of unemployment, $u_t = (\bar{L} - L_t)/\bar{L}$, or on the employment ratio $L_t/\bar{L}$. When $L = 0$, or $u = 1$, the lower bound is zero, then it increases with employment with elasticity γ_1, and at full employment nominal wages cannot be lower than $\gamma_0 W_{t-1}$ as in (2). However, SGU impose the following important assumption on γ_0: $\beta < \gamma_0 \leq \Pi^*$, where β is the subjective discount factor of the representative agent. For simplicity, we also assume $\gamma_0 = \Pi^*$, as SGU do in their quantitative calibration. The DNWR (6) implies the following complementary slackness condition

$$(\bar{L} - L_t)\left[W_t - \gamma_0 (1 - u_t)^{\gamma_1} W_{t-1}\right] = 0 \tag{7}$$

that ties down quite strictly the type of equilibrium under unemployment. If $L_t < \bar{L}$, then in steady state it follows $W_t/W_{t-1} \equiv \Pi^W = \Pi = \gamma_0 (1 - u_t)^{\gamma_1} < \gamma_0 = \Pi^*$. Hence, steady state inflation is below the target whenever there is positive unemployment.

Similar to the previous model, there are accordingly two regimes characterizing the AS in steady state. First, AS is still vertical at the potential output Y^f. Second, the AS^{WR} is upward-sloping in the presence of unemployment due to the binding DNWR constraint:

$$Y_{AS}^{WR} = \left(\frac{\Pi}{\gamma_0}\right)^{\frac{\alpha}{\gamma_1}} Y^f. \tag{8}$$

The two branches of the AS meet at the kink $\Pi = \gamma_0 = \Pi^*$.

The demand side is shaped by a monetary policy rule with a ZLB

$$1 + i_t = \max\left\{1, 1 + i^* + \alpha_\pi (\Pi_t - \Pi^*) + \alpha_y \ln\left(\frac{Y_t}{Y^f}\right)\right\} \tag{9}$$

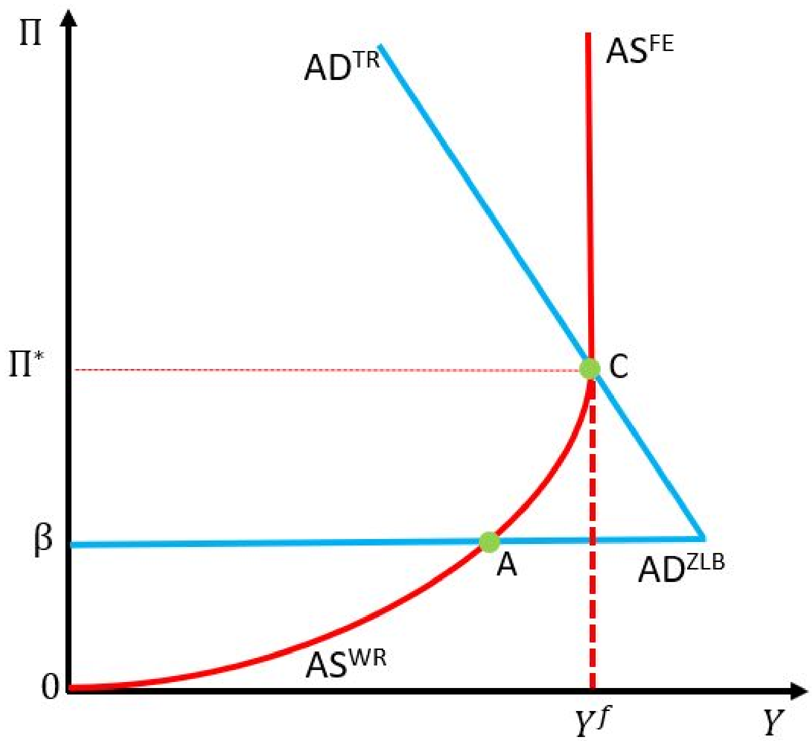
where $1 + i^* = \Pi^*/\beta > 1$. For $1 + i > 1$, equation (9) yields a negative steady state relationship between output and inflation, if monetary policy is active ($\beta\alpha_\pi > 1$), as in EMR model. The main difference between the EMR model and the SGU model lies in the steady state determination of the equilibrium/natural real interest rate. Given the Euler equation, the inverse of β pins down the natural real interest rate in an infinite-life model, so the latter does not depend on the supply and demand of assets as in an OLG model. This implies that, while the AD^{TR} is downward-sloping as in the EMR model, the AD^{ZLB} is now horizontal in this model, rather than upward-sloping. If the ZLB is binding ($i = 0$) and given $1 + r = 1/\beta$, the steady state inflation rate must equal β due to the Fisher equation, whatever the level of steady state output. AD is therefore flat at $\Pi = \beta$, and the AS determines the steady state output for that inflation level.

Figure 4 shows the $AS - AD$ diagram for the SGU model. The assumption in SGU $\beta < \gamma_0 \leq \Pi^*$ guarantees that there does not exist either an intersection between AS^{FE} and AD^{ZLB} or an intersection between AD^{TR} and AS^{WR}. Given these assumptions, there are always two equilibria.[15] As in the previous section, point A is a $ZLB - U$ type of equilibrium, where both the ZLB and the DNWR constraints are binding, and point C is a $TR - FE$ one, where none of the two constraints is binding, the economy is at full employment and inflation at target. However, the $ZLB - U$ equilibrium in the SGU model does not reflect the idea of secular stagnation as described in Summers (2015) that entails $r^f < 0$. By contrast, it is an expectation-driven deflationary equilibrium á la Benhabib et al. (2001).[16] Therefore, we define

[15] There are no restrictions on γ_1. Hence, we can distinguish three cases: if $\gamma_1 > \alpha$, the AS^{WR} is convex as depicted in Figure 4; it is concave for $\gamma_1 < \alpha$; and it is a straight line when $\gamma_1 = \alpha$. Whether the AS^{WR} is convex, concave or a straight line does not affect our results qualitatively, but the $ZLB - U$ equilibrium A is associated with a larger negative output gap when AS^{WR} is concave or a straight line than in the convex case.

[16] Indeed, equilibrium A in Figure 4 is indeterminate, contrary to the corresponding equilibrium in

Figure 4: Steady state equilibria in the SGU model



it as a *deflationary* equilibrium, because $\Pi = \beta < 1$, rather than *secular stagnation* one.

3.1 Dissolving the ZLB equilibrium

We now apply our reflationary income policy to this model by replacing the DNWR (6) with the DNWGR given by (1) and (5). Recall that the idea is to reflate the economy by using the DNWGR constraint to impose *a floor to the rate of growth of nominal wages that depends on the inflation target*. (6) does not do that because wage inflation is bounded by zero when employment is zero. To see how our income

the EMR OLG model.

policy works in this model, we plot the $AS - AD$ diagram for the SGU model with the DNWGR. In this case, the AS^{WR} is flat at the wage/price inflation $\Pi = \delta\Pi^*$ instead of upward-sloping.

Figure 5: Steady state equilibria in the SGU model with DNWGR

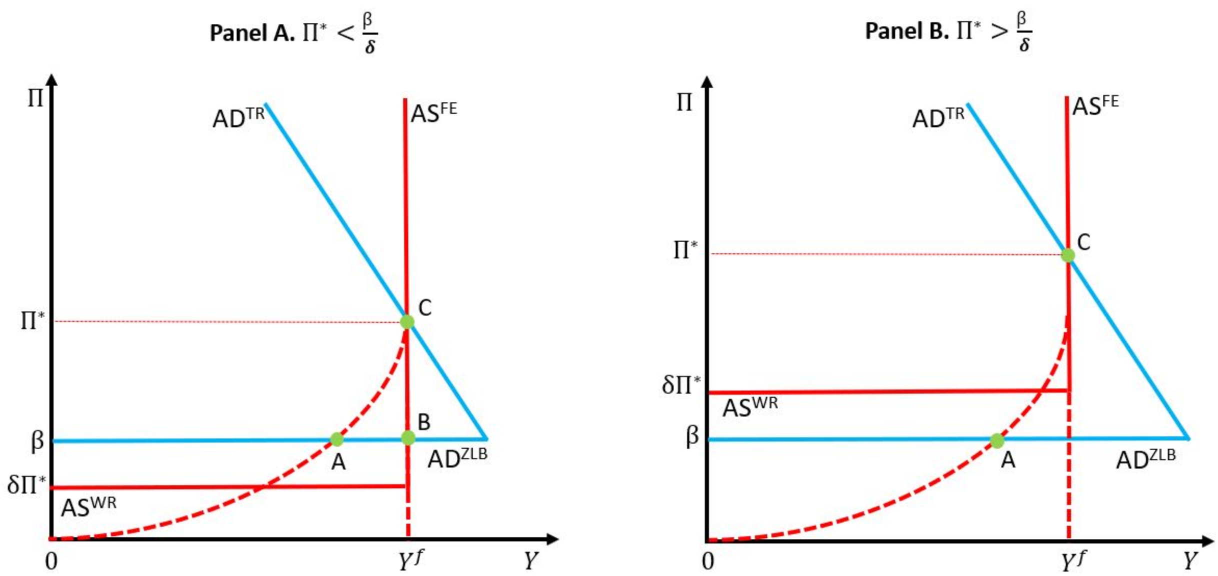


Figure 5 shows how the introduction of the DNWGR yields similar implications as in Section 2. Two are the stark differences compared to the original SGU model. First, if the inflation target is not high enough, $\Pi^* < \beta/\delta$, the presence of the DNWGR makes a new equilibrium to arise at point B (Panel A). This new equilibrium replaces the original deflationary one at point A, and so the economy runs at the potential level even when the ZLB is binding due to deflation. The intuition behind this result is straightforward. Although the current inflation target is not high enough to destroy the ZLB equilibrium, the DNWGR sets a minimum level of wage/price inflation, $\delta\Pi^*$, that is lower than actual inflation, β, allowing the real wage to fall and so stimulating employment. Furthermore, this result resembles that in Proposition 2, and the equilibrium B in Figure 5, though deflationary because of $\Pi = \beta$, resembles the $ZLB - FE$ one in Figure 1. Second, while in the original SGU model two equilibria always exist, for a sufficiently high inflation target, $\Pi^* > \beta/\delta$, deflationary expectations cannot be supported in equilibrium so that the $ZLB - U$ equilibrium A dissolves (Panel B). Intuitively, by forcing the increase in wage inflation above a certain threshold, no level of inflation/deflation supports the ZLB equilibrium. Our DNWGR constraint acts as a coordination device for agents on the now unique $TR - FE$ equilibrium.

We can formalize these results through two propositions that parallel those in the previous section for the OLG model.

Proposition 3. *Assume $\delta \leq 1$. Then, if $\Pi^* > \beta/\delta$, there exists a unique, locally determinate, $TR - FE$ equilibrium, where the ZLB is not binding, the inflation rate is equal to the target and output is at full employment, i.e., $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.*

Proposition 4. *Assume $\delta \leq 1$, and that the economy is trapped in a deflationary $ZLB - U$ equilibrium (point A). Then, the introduction of our DNWGR is always beneficial, in the sense that steady state output increases, irrespective of the economy escapes from deflation.*

The results obtained in the SGU model, confirming those in the EMR model, show the effectiveness of our income policy in tackling the ZLB problem, and its beneficial effect in any case, even when the ZLB is still binding. Notwithstanding, there is an important warning to remember regarding the implementation of our income policy when the liquidity trap is expectation-driven as in SGU.

We check the robustness of our results/propositions to the specification of the DNWGR, assuming a form that includes a fraction of the population with flexible wages in the EMR model (Ascari and Bonchi, 2019), and that minimum wage inflation also depends on unemployment in the SGU model (Appendix A.2.4). In both cases, the main result of the paper regarding the dissolution of the ZLB equilibrium, in Propositions 1 and 3, still holds. This proves the robustness of our policy, not only to the source of the ZLB, but also to the specification of the DNWGR, and so to potential mitigating effects. However, Proposition 4 no longer holds in the SGU model when unemployment affects the DNWGR, because our policy worsens the deflationary equilibrium decreasing output.

The last result emphasizes the difference with the EMR OLG model, in which Proposition 2, and not only Proposition 1, is robust to the specification of the DNWGR. However, the difference between the two models does not depend on the specific form of the DNWGR, but it lies more generally on the demand side, and in particular, on AD^{ZLB}. The latter is upward-sloping, and always steeper than the AS^{WR} in an OLG model because any increase in inflation decreases the real interest rate, spurring demand at the ZLB. In an infinite-life economy, in contrast, the real interest rate is given by $1/\beta$, and so inflation has to $\Pi = \beta$ in a ZLB/deflationary equilibrium. The AS^{WR}, which is upward-sloping when unemployment affects the DNWGR (Figure 9, Appendix A.2.4), like originally in Figure 4, is steeper than the AD^{ZLB}. As both price and wage inflation have to equal β, inflation expectations do not adjust to the intended increase in wage inflation in the ZLB/deflationary equilibrium, and so any attempt to increase wage growth has to be compensated by higher unemployment and lower output. The liquidity trap gets worse because the policy is trying to force an increase in wage inflation, but agents do not believe prices could increase.

Discussion: Policy implementation

While a detailed discussion of policy implementation is outside the scope of the present paper, this is a crucial point that we now briefly discuss. As already said, history provides many examples of successful implementation of income policy through some degree of corporatism to engineer a reduction of inflation.[17] Hence, in some sense, this has already been done "the other way round". Moreover, there are other options available for the government to enforce wage inflation as, for example, the use of profit tax levy or subsidies, and a "comply or explain" policy for firms in the private sector (Wallich and Weintraub, 1971; Okun, 1978; Arbatli et al., 2016).

Although several options are available so that each country can implement reflationary income policies in a way consistent with its economic institutions and industrial relations, we share the opinion of da Silva and Mojon (2019) that the institutional framework of the past disinflationary policies has been proven the most effective, as well as the most well-known. The more recent and negative experience of Japan, which tried to sustain wage inflation within a different institutional arrangement, supports this argument.[18] In particular, we view a national agreement,

[17] The aforementioned case of Italy is not isolated. In March 1983, the Australian government promoted Accord Mark I with the unions to restrain wage increases and so fight high unemployment and inflation. The Accord was renegotiated several times (Accords Mark I-VII) and, as a result of the improvement in industrial relations, a corporatist model emerged. In the 1990s, the Dutch corporatist Polder model gained popularity because of good social and economic performance. It is based on consulting between the government and the social partners, involving them in the design and implementation of socio-economic policies (see Visser and Hemerijck, 1997). Similar models are in place in Belgium and Finland.

[18] The former prime minister of Japan, Shinzo Abe, sought to influence wage negotiations to push for substantial nominal wage increases. However, in contrast with the Italian experience of consultation (i.e., *Concertazione*), the government does not take part directly in the negotiations between the Japan Business Federation (*Keidanren*) and the Trade Union Confederation (*Rengo*), which occur during the "spring offensive" called *Shuntō*. Therefore, wage increases were dismal despite Mr

as in the Italian experience, between employers and union associations and the government capable of making wage indexation to target inflation largely widespread, realizing the switch from DNWR to DNWGR discussed in the models above. The role of the government is central in this arrangement because it strengthens the commitment to raise wage inflation, making it credible also through other policy measures such as raising administrative prices and public-sector nominal wages (Blanchard, 2018; da Silva and Mojon, 2019). On the other hand, the government can strike a balance between unions, which could be more supportive of income policies, and firms, which could raise concerns about the possible adverse impact of wage increases on profitability.[19]

In any case, the agreement and, specifically, the underlying institutional arrangement should not be interpreted necessarily as permanent, implying a radical change in the labor market relations. Indeed, a great advantage of income policies adopted through corporatism is that they are easy to dismantle when no longer necessary, namely when inflation is back to the target. Furthermore, the agreement should establish a *minimum* nominal wage growth rate, in line with targeted inflation, at the national level, leaving further wage increases due to productivity gains to be determined at the firm level. This would allow movements in relative prices across the economy given differences in productivity and to take care of firms' competitiveness in domestic markets.

Abe's call. See, e.g., "Shinzo Abes campaign to raise Japanese wages loses steam", FT online, 22 January 2019. The Japanese government also tried unsuccessfully to incentivize wage increases through tax breaks. See, e.g., "Japan's tax incentive for raising wages may have limited appeal", Reuters, 15 February 2018.

[19] Concretely, the government could trade-off nominal wage increases for corporate tax cuts as suggested by Blanchard and Posen in "Getting serious about wage inflation in Japan", Nikkei Asian Review, 15 December 2015.

Finally, it is worth noting that, though this implementation of the proposed income policy, requires some coordination between the Treasury, which takes part in the wage negotiations, and the central bank, which sets the inflation target, this does not undermine the independence of the monetary authority. Instead, only the explicit support of the central bank, expressed through its communication policy, would reinforce the credibility of a national agreement to sustain wage inflation. Illustrative examples in this sense are the recent disinflationary episodes of Spain in 2010-2012 and Finland in 2016, which occurred with an independent central bank that supported the tripartite agreement between government, trade unions and employers associations publicly.

4 Conclusions

As de-indexing the economy has been proven an effective way to tackle high inflation in past historical episodes, we have investigated whether the same mechanism also works in the opposite direction to engineer inflation at the ZLB. Our policy of "re-indexing" the economy consists in imposing minimum wage inflation that delivers the necessary price inflation to escape from the ZLB.

Specifically, we have studied the ZLB problem through the lens of the EMR OLG model and the SGU infinite-life representative agent model, which both feature a ZLB equilibrium and downwardly rigid nominal wage *level*. Our income policy imposes *a floor on wage inflation that depends on a fraction of the inflation target* through a downward nominal wage *growth* rigidity. This is exactly the opposite of the ceiling on wage inflation imposed in some past disinflationary policies.

Under our assumption, the ZLB equilibrium disappears in both models because the DNWGR acts as a coordination device that destroys the bad ZLB equilibrium. This result is robust to the specification of the DNWGR, and it requires a sufficiently high inflation target.

Our results shed new light on the macroeconomic trends that characterized the pre-COVID 19 era. The massive fiscal and monetary measures implemented in the wake of the Great Recession restored low unemployment rates and sound growth, but they did not bring the economy out of the liquidity trap with inflation at the targeted level. Several explanations have been put forward for this puzzling evidence (see, e.g., Mojon and Ragot, 2019), and our work provides a different "policy" perspective pointing to the "ammunition" saved in the policy packages of the advanced countries: a reflationary income policy. It plays the specific role of re-anchoring inflation expectations and is complementary to fiscal and monetary policy, as already shown in the past disinflationary episodes. According to Bruno (1993), the essence of a "heterodox approach" against high inflation is to promote a coordinated effort through several and different policies. This old lesson appears valid, even with different enemies: low inflation and binding ZLB. On the other hand, our paper provides the theoretical underpinning in support of income policies to reflate economies stuck in long-lasting ZLB episodes with subdued or negative inflation. However, some caveats are in order, along with the further steps to be taken to provide more compelling evidence.

First, although there always exists a minimum level of the inflation target sufficient to escape from the ZLB, this target is compatible with the standard definition of price stability only if the average degree of wage indexation is also high. A tripartite agreement between government, unions, and employers association seems the natural way to promote and enforce indexation clauses. However, the political economy implications of such an agreement deserve further investigation, which goes beyond the scope of the present paper. In particular, the agreement might have more support from unions than capitalists, who might be concerned about the negative impact of wage increases on profitability and international competitiveness. The former point could be particularly important in the case of a monetary union such as the Eurozone, or a currency area in general, where a devaluation to compensate for the higher prices is not possible, and so close coordination among the member states is necessary to implement reflationary income policies and avoid current account imbalances simultaneously.

Second, while the introduction of the DNWGR does not present side effects in a secular stagnation environment, in which any increase in inflation is expansionary, its implementation in an expectation-driven liquidity trap needs to be more accurate. Indeed, if the indexation policy forces the wages to increase by more, but agents do not adjust their inflation expectations upwards, then a deflationary equilibrium still exists, but higher unemployment and lower output could be needed to support it. To further understand the implications of the expectation-driven liquidity trap for the reflationary income policy, we would study the transitional dynamics from the ZLB equilibrium to an equilibrium with full employment and inflation at the target. This, specifically, would require a much richer model with a wider range of nominal rigidities (e.g., price and wage stickiness) and endogenous labor supply to evaluate the costs and benefits of the policy along the transition.

References

Arbatli, Elif, Dennis Botman, Kevin Clinton, Pietro Cova, Vitor Gaspar, Zoltan Jakab, Douglas Laxton, Constant Lonkeng Ngouana, Joannes Mongardini, and Hou Wang, "Reflating Japan; Time to Get Unconventional?," IMF Working Papers 16/157, International Monetary Fund August 2016.

Ascari, Guido and Jacopo Bonchi, "(Dis)Solving the Zero Lower Bound Equilibrium through Income Policy," Working Papers 10/19, Sapienza University of Rome, DISS October 2019.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe, "The Perils of Taylor Rules," *Journal of Economic Theory*, 2001, *96* (1-2), 40–69.

_ , _ , and _ , "Avoiding Liquidity Traps," *Journal of Political Economy*, 2002, *110* (3), 535–563.

Bilbiie, Florin O., "Neo-Fisherian Policies and Liquidity Traps," CEPR Discussion Papers 13334, C.E.P.R. Discussion Papers November 2018.

Blanchard, Olivier, "The Missing Third Leg of the Euro Architecture: National Wage Negotiations," 2018. Remarks delivered at the French TreasuryIMF conference on "Transforming France's economy and completing the integration of the Eurozone", February 15, 2018.

Branten, Eva, Ana Lamo, and Tairi Room, "Nominal wage rigidity in the EU countries before and after the Great Recession: evidence from the WDN surveys," Working Paper Series 2159, European Central Bank June 2018.

Bruno, Michael, *Crisis, Stabilization, and Economic Reform: Therapy by Consensus*, Oxford University Press, 1993.

Cuba-Borda, Pablo and Sanjay R. Singh, "Understanding Persistent Stagnation," Working Papers 1243, Board of Governors of the Federal Reserve System 2019.

da Silva, Luiz A. Pereira and Benoît Mojon, "Exiting low inflation traps by "consensus": nominal wages and price stability," 2019. Speech based on the keynote speech at the Eighth High-level Policy Dialogue between the Eurosystem and Latin American Central Banks, Cartagena de Indias, Colombia, 28-29 November 2019.

Destefanis, Sergio, Giuseppe Mastromatteo, and Giovanni Verga, "Wages and Monetary Policy in Italy Before and After the Wage Agreements," *Rivista Internazionale di Scienze Sociali*, 2005, *113* (2), 289–318.

Dickens, William T., Lorenz Goette, Erica L. Groshen, Steinar Holden, Julián Messina, Mark E. Schweitzer, Jarkko Turunen, and Melanie E. Ward, "How Wages Change: Micro Evidence from the International Wage Flexibility Project," *Journal of Economic Perspectives*, Spring 2007, *21* (2), 195–214.

Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins, "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, January 2019, *11* (1), 1–48.

Fabiani, S., A. Locarno, G. Oneto, and P. Sestito, "Risultati e problemi di un quinquennio di politica dei redditi: una prima valutazione quantitativa," *Rivista*

Internazionale di Scienze Sociali, 1998. Bank of Italy, Temi di Discussione, No. 329.

Fallick, Bruce C., Daniel Villar Vallenas, and William L. Wascher, "Downward Nominal Wage Rigidity in the United States during and after the Great Recession," Working Papers 201602R, Federal Reserve Bank of Cleveland March 2020.

Gressani, Daniela, Luigi Guiso, and Ignazio Visco, "Disinflation in Italy: An analysis with the econometric model of the Bank of Italy," *Journal of Policy Modeling*, 1988, *10* (2), 163–203.

Gust, Christopher, Edward Herbst, David López-Salido, and Matthew E. Smith, "The Empirical Implications of the Interest-Rate Lower Bound," *American Economic Review*, July 2017, *107* (7), 1971–2006.

Holden, Steinar and Fredrik Wulfsberg, "Downward Nominal Wage Rigidity in the OECD," *The B.E. Journal of Macroeconomics*, April 2008, *8* (1), 1–50.

_ **and** _ , "How strong is the macroeconomic case for downward real wage rigidity?," *Journal of Monetary Economics*, May 2009, *56* (4), 605–615.

Krugman, Paul, "The Timidity Trap," *The New York Times*, 2014. March 20, available at https://www.nytimes. com/2014/03/21/opinion/krugman-the-timidity-trap.html.

Kurmann, André and Erika McEntarfer, "Downward Nominal Wage Rigidity in the United States: New Evidence from Worker-Firm Linked Data," School of

Economics Working Paper Series 2019-1, LeBow College of Business, Drexel University January 2019.

Mertens, Karel and Morten O. Ravn, "Fiscal Policy in an Expectations-Driven Liquidity Trap," *Review of Economic Studies*, 2014, *81* (4), 1637–1667.

Messina, Julián and Anna Sanz de Galdeano, "Wage Rigidity and Disinflation in Emerging Countries," *American Economic Journal: Macroeconomics*, January 2014, *6* (1), 102–133.

Mojon, Benoit and Xavier Ragot, "Can an ageing workforce explain low inflation?," BIS Working Papers 776, Bank for International Settlements March 2019.

Okun, Arthur M., "Efficient Disinflationary Policies," *American Economic Review*, May 1978, *68* (2), 348–352.

Schmitt-Grohé, Stephanie and Martín Uribe, "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 2016, *124* (5), 1466–1514.

_ **and** _ , "Liquidity Traps and Jobless Recoveries," *American Economic Journal: Macroeconomics*, 2017, *9* (1), 165204.

Summers, Lawrence H., "Demand Side Secular Stagnation," *American Economic Review*, May 2015, *105* (5), 60–65.

Visser, Jelle and Anton Hemerijck, *'A Dutch miracle': job growth, welfare reform and corporatism in the Netherlands*, Amsterdam: Amsterdam University Press, 1997.

Wallich, Henry C. and Sidney Weintraub, "A Tax-Based Incomes Policy," *Journal of Economic Issues*, June 1971, *5* (2), 1–19.

A Appendix

A.1 Appendix to EMR

A.1.1 Model

The economy consists of overlapping generations of agents who live three periods, firms and a central bank in charge of monetary policy. Population grows at a rate g_t, and there is no capital in the economy. Young households borrow up to an exogenous debt limit D_t by selling a one-period riskless bond to middle-aged households, which supply inelastically their labor endowment $\bar{L}$ for a wage W_t and get the profits Z_t from running a firm. Only middle-aged households work and run a firm. Old agents dissave and consume their remaining wealth. The maximization problem of the representative household is

$$\max_{C^m_{t+1}, C^o_{t+2}} E_t \left\{ \ln C^y_t + \beta \ln C^m_{t+1} + \beta^2 \ln C^o_{t+2} \right\}$$

s.t.

$$C^y_t = B^y_t \tag{A1}$$

$$C^m_{t+1} = Y_{t+1} - (1 + r_t) B^y_t - B^m_{t+1} \tag{A2}$$

$$C^o_{t+2} = (1 + r_{t+1}) B^m_{t+1} \tag{A3}$$

$$(1 + r_t) B^y_t = D_t, \tag{A4}$$

where $Y_t = \frac{W_t}{P_t} L_t + \frac{Z_t}{P_t}$ and labor demand L_t does not necessarily equate labor supply $\bar{L}$, as explained in the main text. C^y_t, C^m_{t+1} and C^o_{t+2} denote the real consumption of the generations, while B^y_t and B^m_{t+1} are respectively the real value of bonds sold by young households and bought by middle-aged ones. Equation (A4) represents the debt limit, which is assumed to be binding for the young generation by imposing $D_{t-1} < \frac{1}{1+(1+\beta)\beta} Y_t$. The optimality condition for the maximization problem is the standard Euler equation

$$\frac{1}{C_t^m} = \beta (1 + r_t) E_t \frac{1}{C_{t+1}^o}. \tag{A5}$$

Generations exchange financial assets in the loan market, and in equilibrium the total amount of funds demanded by young households equals the one supplied by middle-aged ones:

$$(1 + g_t) B_t^y = B_t^m, \tag{A6}$$

where $1 + g_t$ is the ratio between the size of the young and middle generation. We can denote the loan demand on the left-hand side of (A6) with L_t^d and express it

$$L_t^d = \left(\frac{1 + g_t}{1 + r_t} \right) D_t \tag{A7}$$

by using (A4). Combining (A2), (A3), (A4) and (A5) yields the loan supply L_t^s:

$$L_t^s = \frac{\beta}{1 + \beta} (Y_t - D_{t-1}). \tag{A8}$$

The market clearing real interest rate that equates (A7) and (A8) is

$$(1 + r_t) = \frac{(1 + g_t)(1 + \beta) D_t}{\beta (Y_t - D_{t-1})}, \tag{A9}$$

and it coincides with the natural interest rate r_t^f at the potential level of output, $Y_t = Y^f$.

Each middle-aged household runs a firm that is active for just one period in a perfectly competitive market. The production technology of firms exhibits decreasing returns to labor, L_t, which is the only input of production:

$$Y_t = L_t^{\alpha}, \tag{A10}$$

where $0 < \alpha < 1$. Profits are

$$Z_t = P_t Y_t - W_t L_t, \tag{A11}$$

and they are maximized, under the technological constraint (A10), if the real price of labor equals its marginal productivity,

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1}. \tag{A12}$$

Wages are subject to the DNWR constraint (1) that we report again here:

$$W_t = \max\left\{W_t^*, W_t^{flex}\right\}. \tag{A13}$$

The lower bound on the nominal wage, W_t^*, is given by (2) and $W_t^{flex} \equiv \alpha P_t \bar{L}^{\alpha-1}$. Finally, the standard Fisher equation holds:

$$1 + r_t = (1 + i_t) E_t \Pi_{t+1}^{-1}, \tag{A14}$$

where E_t denotes the expectation operator.

A.1.2 Aggregate demand and supply

A competitive equilibrium is a set of quantities $\{C_t^y, C_t^m, C_t^o, B_t^y, B_t^m, Y_t, Z_t, L_t\}$ and prices $\{P_t, W_t, r_t, i_t\}$ that solve (1), (3), (A1), (A2), (A3), (A4), (A5), (A6), (A10), (A11), (A12) and (A14), given $\{D_t, g_t\}$ and initial values for W_{-1} and B_{-1}^m. The steady state equilibrium, in which variables take a constant value, can be represented by aggregate demand and supply.

AS is characterized by two regimes, which depend on equation (1) through the steady state inflation rate. For $\Pi \geq \gamma$, we can derive *AS* from equations (1), (A10) and (A12):

$$Y_{AS}^{FE} = \bar{L}^{\alpha} = Y^f.$$

Otherwise, the aggregate supply is given by

$$\Pi = \gamma.$$

The regime of *AD* depends on the lower bound on the nominal interest rate expressed in equation (3). For a positive nominal interest rate, $1+i > 1$, we get the following *AD*, which relates negatively inflation to output, by combining equations (3), (A9) and (A14):

$$Y_{AD}^{TR} = D + \left(\frac{1+\beta}{\beta}\right)\left(\frac{1+g}{1+r^f}\right)\left(\frac{\Pi^*}{\Pi}\right)^{\phi_\pi - 1} D.$$

If the inflation rate is higher than the target, the nominal interest rate increases more than inflation ($\phi_\pi > 1$), resulting in a higher real interest rate that increases savings and contracts demand. We derive a different *AD* from the equations above, when the ZLB is binding:

$$Y_{AD}^{ZLB} = D + \left(\frac{1+\beta}{\beta}\right)(1+g)\,\Pi D.$$

At the ZLB, the AD defines a positive relationship between steady state inflation and output, because higher inflation reduces the real interest rate through the Fisher equation, $1+r = 1/\Pi$, with a consequent boost in aggregate demand. We compute the inflation rate at which the ZLB becomes binding from the two arguments on the right-hand side of (3):

$$\Pi^{kink} = \left[\frac{1}{(1+r^f)}\right]^{\frac{1}{\phi\pi}} \Pi^{*\frac{\phi\pi-1}{\phi\pi}}.$$

A.1.3 Proof of Proposition 1

We study here the calibrations of the inflation target associated with the five panels in Figure 3, and we distinguish the general case $\delta < 1$, analyzed in that figure, from the limiting case $\delta = 1$. Before turning to the proof of Proposition 1, which is explained for $\delta < 1$ but applies also to $\delta = 1$, we describe the three possible steady state equilibria in the EMR model (see Figure 1), considering now DNWGR (5) instead of DNWR (2):

(A) $ZLB-U$ that occurs at the intersection of the AD^{ZLB} and the AS^{WR}, and it features

$$Y = D + \left(\frac{1+\beta}{\beta}\right)(1+g)\,\delta\Pi^{*}D < Y^f$$

$$i = 0$$

$$\Pi = \delta\Pi^* \leq \Pi^*;$$

(B) $ZLB - FE$ that occurs at the intersection of the AD^{ZLB} and the AS^{FE}, and it features

$$Y = Y^f$$

$$i = 0$$

$$1 < \Pi = \frac{1}{1+r^f} < \Pi^*;$$

(C) $TR - FE$ that occurs at the intersection of the AD^{TR} and the AS^{FE}, and it features

$$Y = Y^f$$

$$i > 0$$

$$\Pi = \Pi^*.$$

- **Case** $\delta < 1$.

  As the level of the inflation target increases, five different cases, two of which are non generic emerge in Figure 3: Panel A: if $\Pi^* < 1/(1+r^f)$, only the $ZLB - U$ equilibrium exists at point A; Panel B: if $\Pi^* = \frac{1}{1+r^f}$, two equilibria exist: a $ZLB - U$ equilibrium at point A and an equilibrium at point B/C that is a combination between $ZLB - FE$ and $TR - FE$, where output is at full employment, the nominal interest rate prescribed by the Taylor rule is exactly zero and the inflation rate is equal to the target; Panel C: if $\frac{1}{1+r^f} < \Pi^* < \frac{1}{\delta(1+r^f)}$, three equilibria exist: $ZLB - U$ at point A, $ZLB - FE$ at point

B and $TR-FE$ at point C; Panel D: if $\Pi^{*}=\frac{1}{\delta(1+r^{f})}$, two equilibria exist: $TR-FE$ at point C and an equilibrium at point A/B that is a combination between $ZLB-U$ and $ZLB-FE$, where output is at full employment, the ZLB is binding (and i is off the Taylor rule) and the inflation rate is $\Pi=\delta\Pi^{*}$; Panel E: if $\Pi^{*}>\frac{1}{\delta(1+r^{f})}$, only the $TR-FE$ equilibrium exists at point C. We start our analysis of these five cases from the first and the last panel, which imply a unique equilibrium. Then, we derive the other cases. A proof of the Proposition 1 follows from the analysis of the case $\Pi^{*}>\frac{1}{\delta(1+r^{f})}$.

Panel A. $\Pi^{*}<\frac{1}{1+r^{f}}$. The second term in the max operator of equation (3) is lower than 1 for $\Pi=\Pi^{*}$, so $i=0$ and a $TR-FE$ equilibrium is impossible. As the resulting inflation level is $\Pi<\Pi^{*}<\frac{1}{1+r^{f}}$ because of the ZLB, even a $ZLB-FE$ equilibrium cannot exist and the only possible equilibrium is of the type $ZLB-U$.

Panel E. $\Pi^{*}>\frac{1}{\delta(1+r^{f})}$. Even if the inflation level reaches its lower bound $\Pi=\delta\Pi^{*}$, $r=r^{f}$ (and so $Y=Y^{f}$) can be achieved without hitting the ZLB. This can be verified by substituting r for r^{f} and Π for $\delta\Pi^{*}$ in the Fisher equation (A14). As the ZLB is not binding ($i>0$), $ZLB-U$ and $ZLB-FE$ equilibria cannot emerge and the unique equilibrium is of the type $TR-FE$.

Panel B. $\Pi^{*}=\frac{1}{1+r^{f}}$. There exists an equilibrium with inflation at the target and output at the potential in this case. In fact, the term $\left(1+r^{f}\right)\Pi^{*}\left(\frac{\Pi}{\Pi^{*}}\right)^{\phi_{\pi}}$ in equation (3) is 1 for $\Pi=\Pi^{*}$. This equilibrium features accordingly $Y=Y^{f}$ (because $r=r^{f}$), $i=0$ and $\Pi=\Pi^{*}=\frac{1}{1+r^{f}}$, so it is a combination between

$ZLB-FE$ and $TR-FE$ equilibria. However, this is not the unique equilibrium, but there still exists a $ZLB-U$ equilibrium because $\Pi^* < \frac{1}{\delta\left(1+r^f\right)}$.

Panel C. $\frac{1}{1+r^f} < \Pi^* < \frac{1}{\delta\left(1+r^f\right)}$. Given $\frac{1}{1+r^f} < \Pi^*$, the second term in the max operator of the Taylor rule (3) is greater than 1 for $\Pi = \Pi^*$, so the ZLB is not binding in correspondence of the inflation target and the natural interest rate. As a consequence, a $TR-FE$ equilibrium exists, but it is not the unique equilibrium given that $\Pi^* < \frac{1}{\delta\left(1+r^f\right)}$. Even $ZLB-FE$ and $ZLB-U$ equilibria emerge and, in particular, the $ZLB-FE$ equilibrium differs from the type $TR-FE$ ($\frac{1}{1+r^f} = \Pi < \Pi^*$).

Panel D. $\frac{1}{1+r^f} < \Pi^* = \frac{1}{\delta\left(1+r^f\right)}$. For $\Pi = \delta\Pi^*$ and $r^f = r$, the Fisher equation (A14) implies binding ZLB ($i = 0$). So, even if the DNWGR constraint bites, for a zero nominal interest rate is possible to achieve $Y = Y^f$. This means that, along with a $TR-FE$ equilibrium that still exists because $\Pi^* > \frac{1}{1+r^f}$, an equilibrium with binding ZLB survives. Given $i = 0$, it follows from the Fisher equation

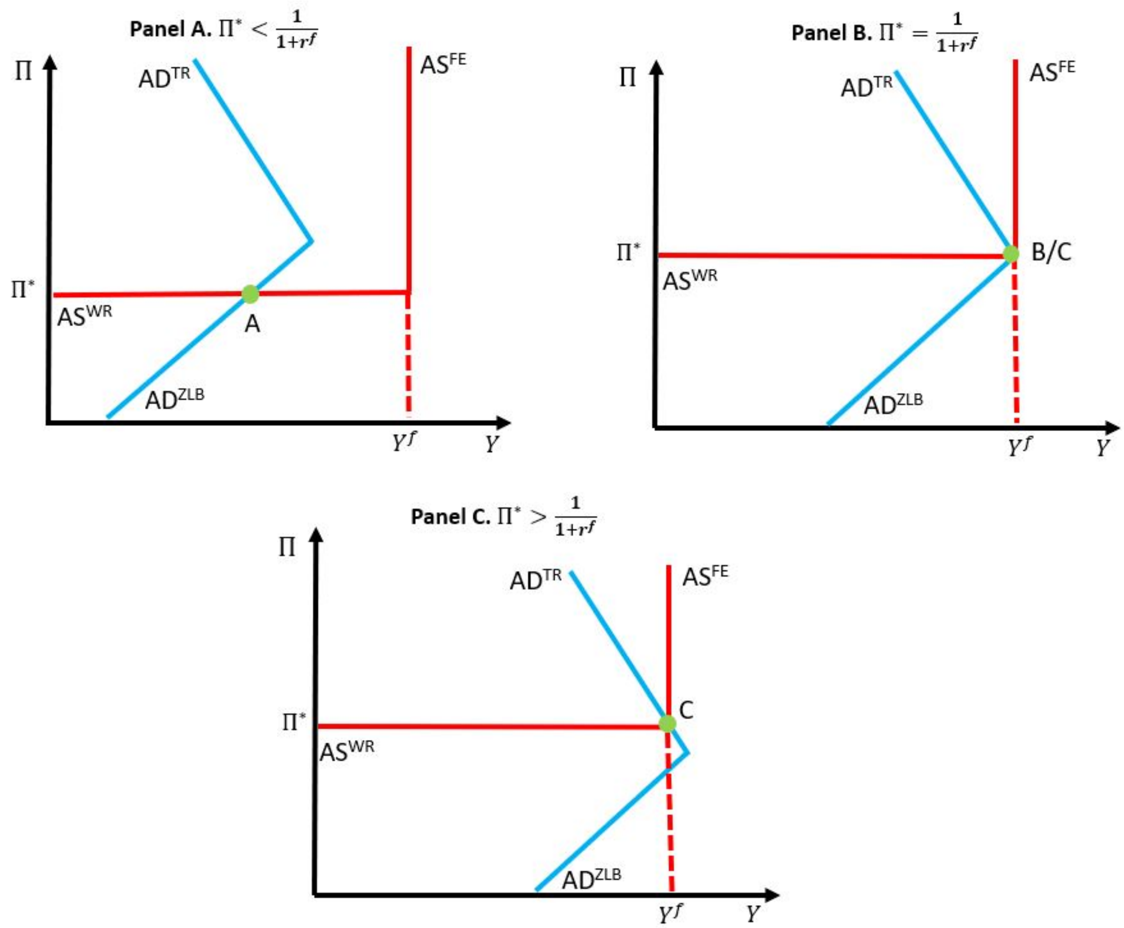
$$\Pi = \delta\Pi^* = \frac{1}{1+r^f}.$$

Therefore, this equilibrium is a combination between $ZLB-U$ and $ZLB-FE$ equilibria.

- **Case $\delta = 1$.**

For $\delta = 1$, only three cases emerge (Figure 6). While Panel A is unchanged, Panel B and D of Figure 3 merge in Panel B of Figure 6, where the $ZLB-U$ equilibrium does not arise and the unique equilibrium is a combination

Figure 6: Steady state equilibria in the EMR model with DNWGR ($\delta = 1$)



between a $ZLB - FE$ and a $TR - FE$ equilibrium ($Y = Y^f$, $i = 0$, and $\Pi = \Pi^* = \frac{1}{1+r^f}$). On the other hand, Panel C of Figure 3 disappears, and Panel E of the same figure collapses to Panel C in Figure 6.

A.1.4 Fiscal policy

We introduce fiscal policy, along with our reflationary income policy, in the EMR model. Apart from fiscal policy, the model is the same as outlined in Appendix

A.1.1, but (5) replaces (2). Moreover, the level of public debt is not sufficiently high to make the natural interest rate positive; otherwise, the secular stagnation equilibrium would not exist.

A fiscal policy regime is given by specific values for $(T_t^y, T_t^m, T_t^o, G_t, B_t^g)$. T_t^i denotes lump-sum taxes levied on the generations, with $i = y, m, o$. Taxation, along with government debt in the form of one-period bonds, B_t^g,[20] finances public expenditure G_t and debt repayment, as formalized by the government budget constraint

$$(1+g_t)T_t^y + T_t^m + \frac{T_t^o}{(1+g_t)} + B_t^g = G_t + \frac{(1+r_{t-1})}{(1+g_t)} B_{t-1}^g, \tag{A15}$$

which is normalized in terms of the size of the middle generation. The introduction of fiscal policy alters the household's budget constraints, which become

$$C_t^y + T_t^y = B_t^y, \tag{A16}$$

$$C_{t+1}^m = Y_{t+1} - T_{t+1}^m - (1+r_t)B_t^y - B_{t+1}^m, \tag{A17}$$

$$C_{t+2}^o = (1+r_{t+1})B_{t+1}^m - T_{t+2}^o, \tag{A18}$$

without changing the Euler equation. However, the relations governing the loan market change. The loan demand, taken at the steady state, is

$$L^d = \left(\frac{1+g}{1+r}\right) D + B^g, \tag{A19}$$

[20]Ricardian equivalence does not hold, as long as middle-aged households view an increase in public debt as permanent, namely they do not expect higher taxes next period.

and the loan supply is

$$L^s = \frac{\beta}{1+\beta}\left(Y - D - T^m\right) + \frac{T^o}{\left(1+\beta\right)\left(1+r\right)}. \tag{A20}$$

As a consequence, the natural interest rate, which equals demand and supply for loans at the potential level of output Y^f, is now given by

$$1 + r^f = \frac{\left(1+\beta\right)\left(1+g\right)D - T^o}{\beta\left(Y^f - D - T^m\right) - \left(1+\beta\right)B^g}. \tag{A21}$$

Furthermore, we can express the condition for the existence of a unique $TR - FE$ equilibrium, defined in Proposition 1, as

$$\Pi^* > \frac{1}{\delta(1+r^f)} = \frac{\beta\left(Y^f - D - T^m\right) - \left(1+\beta\right)B^g}{\delta\left[\left(1+\beta\right)\left(1+g\right)D - T^o\right].}$$

The complementarity between fiscal policy and our income policy is evident from the expression above. The higher the level of public debt, the higher the natural interest rate and so the lower the minimum inflation target to make the $TR - FE$ equilibrium unique. On the other hand, the higher the inflation target, the lower the natural interest rate and the public debt compatible with a unique $TR - FE$ equilibrium.

A.1.5 DNWR and DNWGR

The model is the same as sketched in Section 2 and outlined in more detail in Appendix A.1.1, except for a different specification of the wage rigidity:

$$W_t = \max\left\{\gamma^{WL} W_{t-1} + \gamma^{WG}\Pi^* W_{t-1} + \left(1-\gamma^{WL}-\gamma^{WG}\right) W_t^{flex}, W_t^{flex}\right\}, \quad \text{(A22)}$$

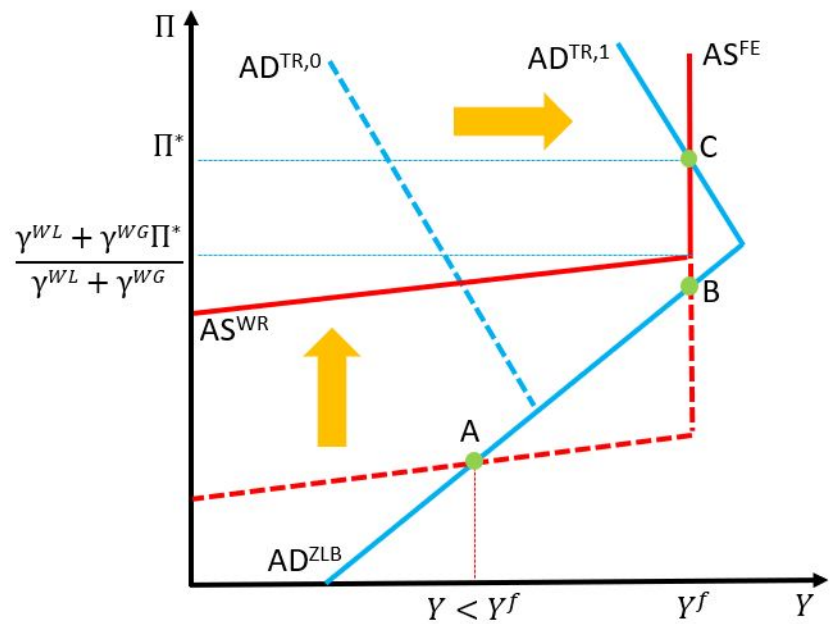
where $W_t^{flex} \equiv \alpha P_t \bar{L}^{\alpha-1}$, $0 \leq \gamma^{WL} \leq 1$ and $0 \leq \gamma^{WG} \leq 1$. It accounts for a "mixed" regime of wage rigidity in which DNWR and DNWGR coexist so that there exists both a downward rigidity to the wage level measured by γ^{WL} and to the wage growth measured by γ^{WG}. When wage rigidities are at work, DNWR prevents nominal wage cuts for a share γ^{WL} of workers, DNWGR prevents wage growth from falling below Π^* for a share γ^{WG} of workers, and the remaining share has flexible wages. Instead, if downward wage rigities are not binding, wages of all workers are flexible.

The wage rigidity assumed alters the shape of AS. While for $\Pi \geq \frac{\gamma^{WL}+\gamma^{WG}\Pi^*}{\gamma^{WL}+\gamma^{WG}}$ aggregate supply is still given by $Y_{AS}^{FE} = Y^f$, it becomes

$$Y_{AS}^{WR} = \left[\frac{1-\left(\gamma^{WL}+\gamma^{WG}\Pi^*\right)\frac{1}{\Pi}}{1-\gamma^{WL}-\gamma^{WG}}\right]^{\frac{\alpha}{1-\alpha}} Y^f \quad \text{(A23)}$$

for $\Pi < \frac{\gamma^{WL}+\gamma^{WG}\Pi^*}{\gamma^{WL}+\gamma^{WG}}$. We compute this equation from (A10), (A12) and (A22), and we depict it as an upward-sloping curve in Figure 7. If inflation falls below $\frac{\gamma^{WL}+\gamma^{WG}\Pi^*}{\gamma^{WL}+\gamma^{WG}}$, wages cannot adjust downward because of both DNWR and DNWGR, and the resulting involuntary unemployment causes output to be lower than the potential. This leads to a positive relationship between inflation and output that fol-

Figure 7: Unique steady state equilibrium with DNWR and DNWGR



lows from a too high real wage: as inflation increases, wage rigidities bite less so that the real wage approaches the level consistent with full employment, reducing the output gap.

Although the segment of the AS corresponding to binding wage rigidity is now upward-sloping, not flat like in Section 2, the central mechanism behind our result still holds (Figure 7). Even if DNWR and DNWGR coexist, the AS curve moves with the inflation target because its "kink", $\frac{\gamma^{WL}+\gamma^{WG}\Pi^*}{\gamma^{WL}+\gamma^{WG}}$, depends on Π^*. Hence, raising Π^* shifts the AS^{WR} upward. We can accordingly establish a proposition similar to Proposition 1 in Section 2 and Proposition 2 continues to hold.

Proposition 5. *Assume $r^f < 0$, $0 \leq \gamma^{WL} \leq 1$, and $0 \leq \gamma^{WG} \leq 1$. Then, if $\Pi^* > \frac{1}{1+r^f} + \left(\frac{1}{1+r^f} - 1\right)\frac{\gamma^{WL}}{\gamma^{WG}}$, there exists a unique, locally determinate, $TR-FE$ equilibrium, where the ZLB is not binding, the inflation rate is equal to the target and output is* *at full employment, i.e.,* $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.

Before turning to the proof of the proposition, it is worth discussing briefly the condition for the existence of a unique $TR-FE$ equilibrium. This condition shows that the higher the degree of wage indexation *per se*, γ^{WG}, and relative to that of DNWR, $\frac{\gamma^{WL}}{\gamma^{WG}}$, the lower the inflation target necessary to destroy the *ZLB-U* equilibrium.

Proof:

There are three possible steady state equilibria in the EMR OLG model with wage rigidity (A22):

(A) $ZLB-U$ that occurs at the intersection of the AD^{ZLB} and the AS^{WR}, and it features

$$Y = \left[\frac{1 - \left(\gamma^{WL} + \gamma^{WG}\Pi^*\right)\frac{1}{\Pi}}{1 - \gamma^{WL} - \gamma^{WG}} \right]^{\frac{\alpha}{1-\alpha}} Y^f$$

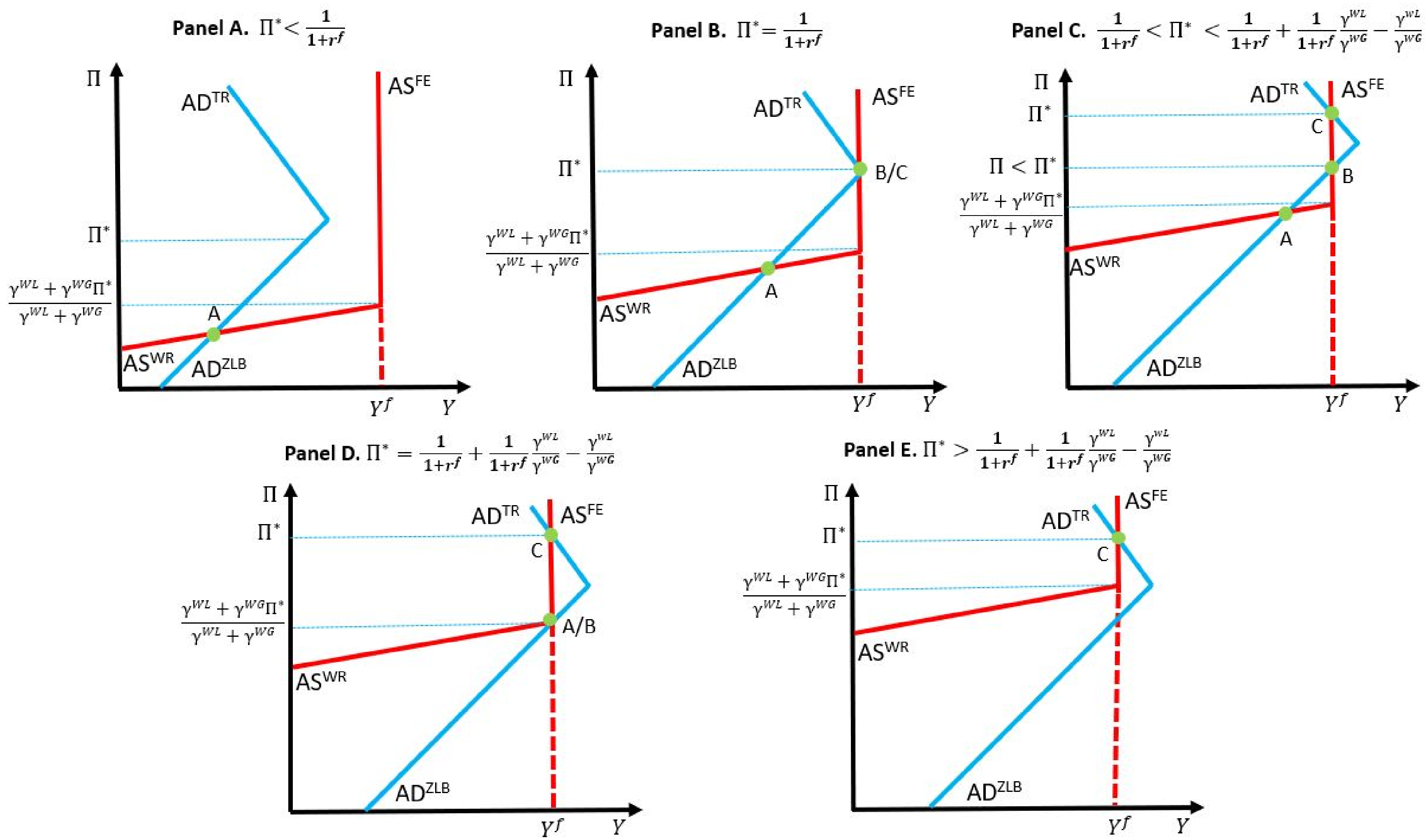
$$i = 0$$

$$\Pi = \frac{1}{1+r} < \Pi^*$$

(B) $ZLB-FE$ that is identical to the equilibrium in the proof of Proposition 1;

(C) $TR-FE$ that is identical to the equilibrium in the proof of Proposition 1.

Figure 8: Steady state equilibria in the EMR model with DNWR and DNWGR



If $r^f < 0$, as the level of the inflation target increases, five different cases can emerge, and we depict them in Figure 8. These cases and the corresponding equilibria are identical to those in Figure 3, described in Appendix A.1.3 (case $\delta < 1$). The only difference, regarding Panels C, D and E, is the condition associated with the case under investigation, which reflects the different assumption about wage rigidity. Now, we study the parameterizations of Π^* corresponding to these five cases. The structure of the proof is the same as in Appendix A.1.3: we first explain Panel A and E, which feature a unique equilibrium, then we study Panel B, C, and D featuring multiple equilibria. A proof of Proposition 5 follows from the analysis of the case $\Pi^* > \frac{1}{1+r^f} + \left(\frac{1}{1+r^f} - 1\right)\frac{\gamma^{WL}}{\gamma^{WG}}$.

Panel A. $\Pi^* < \frac{1}{1+r^f}$. The proof is the same as in Proposition 1.

Panel E. $\Pi^* > \frac{1}{1+r^f} + \left(\frac{1}{1+r^f} - 1\right)\frac{\gamma^{WL}}{\gamma^{WG}}$. Even if inflation reaches the level $\Pi = \frac{\gamma^{WL}+\gamma^{WG}\Pi^*}{\gamma^{WL}+\gamma^{WG}}$ and so wage rigidities start to bind, the real interest rate can equate its natural level, and so output can equate its potential level, without hitting the ZLB, $i > 0$. We can verify this by substituting r for r^f and Π for $\frac{\gamma^{WL}+\gamma^{WG}\Pi^*}{\gamma^{WL}+\gamma^{WG}}$, in the Fisher equation (A14). Given $i > 0$, $ZLB-U$ and $ZLB-FE$ equilibria cannot emerge and the unique equilibrium is of the type $TR-FE$.

Panel B. $\Pi^* = \frac{1}{1+r^f}$. In this case, the term $\left(1+r^f\right)\Pi^*\left(\frac{\Pi}{\Pi^*}\right)^{\phi_\pi}$ in equation (3) is 1 for $\Pi = \Pi^*$. There exists accordingly an equilibrium featuring $Y = Y^f$ (because $r = r^f$), $i = 0$ and $\Pi = \Pi^* = \frac{1}{1+r^f}$. This equilibrium is a combination between $ZLB-FE$ and $TR-FE$ equilibria, but it is not the unique equilibrium because there still exists a $ZLB-U$ equilibrium, given $\Pi^* < \frac{1}{1+r^f} + \left(\frac{1}{1+r^f} - 1\right)\frac{\gamma^{WL}}{\gamma^{WG}}$.

Panel C. $\frac{1}{1+r^f} < \Pi^* < \frac{1}{1+r^f} + \left(\frac{1}{1+r^f} - 1\right)\frac{\gamma^{WL}}{\gamma^{WG}}$. Given $\frac{1}{1+r^f} < \Pi^*$, the second term in the max operator of the Taylor rule (3) is greater than 1 for $\Pi = \Pi^*$, so the ZLB is not binding in correspondence to the inflation target and the natural interest rate. Although a $TR - FE$ equilibrium exists, this is not the unique equilibrium given that $\Pi^* < \frac{1}{1+r^f} + \left(\frac{1}{1+r^f} - 1\right)\frac{\gamma^{WL}}{\gamma^{WG}}$. Even $ZLB - FE$ and $ZLB - U$ equilibria emerge and, in particular, the $ZLB - FE$ equilibrium distinguishes starkly from the type $TR - FE$ because $\frac{1}{1+r^f} = \Pi < \Pi^*$.

Panel D. $\frac{1}{1+r^f} < \Pi^* = \frac{1}{1+r^f} + \left(\frac{1}{1+r^f} - 1\right)\frac{\gamma^{WL}}{\gamma^{WG}}$. For $\Pi = \frac{\gamma^{WL}+\gamma^{WG}\Pi^*}{\gamma^{WL}+\gamma^{WG}}$ and $r^f = r$, the Fisher equation (A14) implies $i = 0$. Even if the wage rigities are at work, for a zero nominal interest rate it is possible to achieve $Y = Y^f$. Hence, along with a $TR - FE$ equilibrium that always emerges for $\Pi^* > \frac{1}{1+r^f}$, an equilibrium with binding ZLB survives. This second equilibrium is a combination between $ZLB - U$ and $ZLB - FE$ equilibria because, given $i = 0$, we get

$$\Pi = \frac{\gamma^{WL} + \gamma^{WG}\Pi^*}{\gamma^{WL} + \gamma^{WG}} = \frac{1}{1+r} = \frac{1}{1+r^f}.$$

from the Fisher equation.

A.2 Appendix to SGU

A.2.1 Model

Unless otherwise mentioned, the notation is identical to that of the model in Appendix A.1.1. The representative household seeks to maximize the utility function

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} \right)$$

subject to the constraints

$$P_t C_t + B_t = W_t L_t + Z_t + (1 + i_{t-1}) B_{t-1}$$

$$lim_{j\to\infty} E_t \left[\prod_{s=0}^{j} (1 + i_{t+s})^{-1} \right] B_{t+j+1} \geq 0.$$

We assume $\sigma = 1$ so that the utility function is logarithmic. C_t denotes the real consumption expenditure, and B_t is the value of risk-free bonds in nominal terms. The optimality conditions for the household's problem is the Euler equation

$$C_t^{-1} = \beta \left(1 + i_t\right) E_t \left(\frac{C_{t+1}^{-1}}{\Pi_{t+1}} \right) \tag{A24}$$

and the no-Ponzi-game constraint

$$lim_{j\to\infty} E_t \left[\prod_{s=0}^{j} (1 + i_{t+s})^{-1} \right] B_{t+j+1} = 0,$$

which holds with equality. The problem of the representative firm is the same as illustrated in Appendix A.1.1, and the DNWR described in the main text is

$$W_t \geq \gamma_0 \left(\frac{L}{\bar{\bar{L}}} \right)^{\gamma_1} W_{t-1}.$$

The aggregate resource constraint imposes

$$Y_t = C_t \tag{A25}$$

and the aggregate rate of unemployment is

$$u_t = \frac{\bar{L} - L_t}{\bar{L}}. \tag{A26}$$

A.2.2 Aggregate demand and supply

A competitive equilibrium is a set of processes $\{Y_t, C_t, L_t, u_t, \Pi_t, W_t, i_t\}$ that solve (6), (7), (9), (A10), (A12), (A24), (A25) and (A26), given the initial value for W_{-1}. Before defining the steady state equilibrium in the SGU model, we study aggregate demand and supply, which are characterized by two regimes, in steady state. For $\Pi \geq \gamma_0 = \Pi^*$, AS is obtained from (6), (7), and (A10):

$$Y_{AS}^{FE} = Y^f.$$

By combining the same equations, AS becomes

$$Y_{AS}^{WR} = \left(\frac{\Pi}{\gamma_0}\right)^{\frac{\alpha}{\gamma_1}} Y^f$$

when $\Pi < \gamma_0 = \Pi^*$. Now, we turn to aggregate demand. For a positive nominal interest rate,

$$1 + i = \frac{\Pi^*}{\beta} + \alpha_\pi (\Pi - \Pi^*) + \alpha_y \ln\left(\frac{Y}{Y^f}\right) > 1,$$

we can compute AD from the Taylor rule by substituting $1+i$ for its steady state value $\frac{\Pi}{\beta}$:

$$\ln Y_{AD}^{TR} = \ln Y^f - \frac{\beta\alpha_\pi - 1}{\beta\alpha_y}\left(\Pi - \Pi^*\right).$$

It can be alternatively expressed as:

$$Y_{AD}^{TR} = \frac{Y^f}{e^{\Phi(\Pi - \Pi^*)}}$$

where $\Phi = \frac{(\beta\alpha_\pi - 1)}{\beta\alpha_y}$. If the ZLB binds, $1+i=1$, AD becomes

$$\Pi = \beta$$

and it is computed by following the same steps as above.

A.2.3 Steady State Equilibrium

Given $\beta < \gamma_0 = \Pi^*$, there are two possible steady state equilibria in the model of SGU with DNWR:

(A) $ZLB - U$ that occurs at the intersection of the AD^{ZLB} and the AS^{WR}, and it features

$$Y = C = \left(\frac{\beta}{\gamma_0}\right)^{\frac{\alpha}{\gamma_1}} Y^f < Y^f$$

$$i = 0$$

$$\Pi = \beta$$

(C) $TR - FE$ that occurs at the intersection of the AD^{TR} and the AS^{FE}, and it features

$$Y = C = Y^f$$

$$i = \frac{\Pi^*}{\beta} - 1 > 0$$

$$\Pi = \Pi^*$$

An additional equilibrium emerges when our DNWGR operates:

(B) deflationary $ZLB - FE$ that occurs at the intersection of the AD^{ZLB} and the AS^{FE}, and it features:

$$Y = C = Y^f$$

$$i = 0$$

$$\Pi = \beta$$

A.2.4 Different specification of the DNWGR

We specify a different DNWGR, in place of (5), in which the minimum wage inflation depends on unemployment, like the minimum wage level in equation (6):

$$\frac{W_t}{W_{t-1}} \geq \delta\Pi^* + \gamma(u_t) = \delta\Pi^* + \gamma_0 (1 - u_t)^{\gamma_1} . \tag{A27}$$

We now assume that $\beta < \delta\Pi^* + \gamma_0 \leq \Pi^*$ (and so $\delta < 1$), which is the equivalent assumption to $\beta < \gamma_0 \leq \Pi^*$ in the SGU case. Accordingly, the AS^{WR} becomes

$$Y_{AS}^{WR} = \left(\frac{\Pi - \delta\Pi^*}{\gamma_0} \right)^{\frac{\alpha}{\gamma_1}} Y^f . \tag{A28}$$

Figure 9: Steady state equilibria in the SGU model with different DNWGR

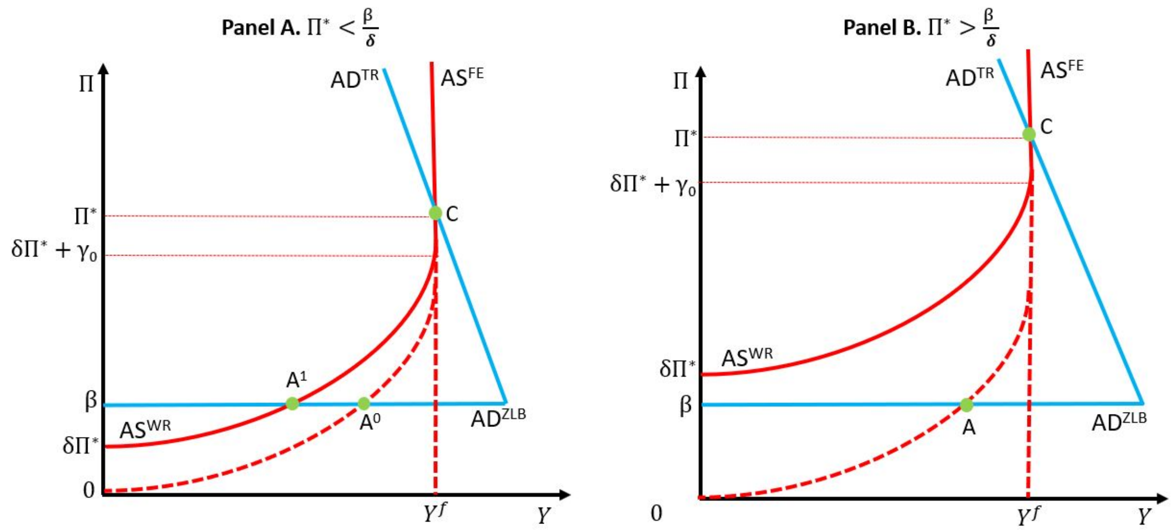


Figure 9 shows how this modification to the DNWGR yields similar, but not identical, implications as in the standard specification (5). Panel B is substantially unchanged compared to the same panel of Figure 5 corresponding to the standard specification. For a sufficiently high inflation target such that $\Pi^* > \beta/\delta$ deflationary expectations cannot be supported in equilibrium, and so the $ZLB-U$ equilibrium A disappears, leaving as unique equilibrium the $TR-FE$ one at point C. On the other hand, Panel A displays a remarkable difference if compared to the same panel of Figure 5. When $\Pi^* < \beta/\delta$, the introduction of the DNWGR no longer improves the deflationary equilibrium causing full employment even at the ZLB. On the contrary, it worsens the deflationary equilibrium reducing output for the same level of deflation, $\Pi = \beta$. Indeed, it is not a deflationary $ZLB-FE$ equilibrium, like B in Figure 5, to replace the original $ZLB-U$ equilibrium A^0, but another equilibrium of the same kind featuring lower output, A^1.

We conclude by stating two propositions summarizing the results above.

Proposition 6. *Assume* $\beta < \delta\Pi^* + \gamma_0 \leq \Pi^*$. *Then, if* $\Pi^* > \beta/\delta$, *there exists a unique, locally determinate,* $TR - FE$ *equilibrium, where the ZLB is not binding, the inflation rate is equal to the target and output is at full employment, i.e.,* $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.

Proposition 7. *Assume* $\beta < \delta\Pi^* + \gamma_0 \leq \Pi^*$ *and that the economy is trapped in a deflationary equilibrium,* $ZLB - U$ *(Panel A). Then, the introduction of our DNWGR is always detrimental, in the sense that steady state output decreases in a ZLB equilibrium, unless this increase is sufficient to escape from deflation.*